Weekly Assignment #9

MAT 229, Spring 2021

Instructions: **Show your work!**

1. Ratio test (and fall-backs)

a. For which of the following series does the ratio test gives an answer about the series's convergence? For these series, give that answer.

- $\sum_{k=1}^{\infty} \frac{2}{k^3+1}$

```
In[249]:= b[k_] := 2 / (k^3 + 1)
          Limit[Abs[b[k + 1] / b[k]], k → Infinity]
Out[250]= 1
```

inconclusive

$$\lim_{k\to\infty}\left|\frac{\frac{2}{(k+1)^3+1}}{\frac{2}{k^3+1}}\right| = \lim_{k\to\infty}\frac{k^3+1}{(k+1)^3+1} = 1$$

(limit of rational expression)

- $\sum_{k=1}^{\infty} \frac{1}{k\,3^k}$

```
In[251]:= b[k_] := 1 / (k 3^k)
          Limit[Abs[b[k + 1] / b[k]], k → Infinity]
Out[252]= 1/3
```

convergent

$$\lim_{k\to\infty}\left|\frac{\frac{1}{(k+1)3^{k+1}}}{\frac{1}{k3^k}}\right| = \lim_{k\to\infty}\left|\frac{k}{3(k+1)}\right| = 1/3$$

- $\sum_{k=1}^{\infty} \frac{(-2)^k}{k^{10}}$

```
In[253]:= b[k_] := (-2)^k / (k^10)
          Limit[Abs[b[k + 1] / b[k]], k → Infinity]
Out[254]= 2
```

di

$$\lim_{k\to\infty}\left|\frac{\frac{(-2)^{k+1}}{(k+1)^{10}}}{\frac{(-2)^k}{k^{10}}}\right| = \lim_{k\to\infty}\left|\frac{2k^{10}}{(k+1)^{10}}\right| = 2$$

- $\sum_{k=1}^{\infty} \frac{k}{1+3k}$

```
In[255]:= b[k_] := k / (1 + 3 k)
          Limit[Abs[b[k + 1] / b[k]], k → Infinity]
Out[256]= 1
```

$$\lim_{k\to\infty}\left|\frac{\frac{k+1}{1+3(k+1)}}{\frac{k}{1+3k}}\right| = \lim_{k\to\infty}\frac{k+1}{k}\cdot\frac{1+3k}{1+3(k+1)} = 1$$

b. For any of the series in the first part for which the ratio test was inconclusive, use another test to determine if the series converges or diverges.

$$S = \sum_{k=1}^{\infty} \frac{2}{k^3+1} < \sum_{k=1}^{\infty}\frac{2}{k^3} = 2\underbrace{\sum_{k=1}^{\infty}\frac{1}{k^3}}$$

convergent p-series

critical that it be <

∴ S is convergent.

```
b[k_] := 2/(k^3+1)
a[k_] := 1/(k^3)
Limit[Abs[b[k]/a[k]], k → Infinity]
(* Converges by comparison with a convergent p-series. *)
```

Out[263]= 2

- $\sum_{k=1}^{\infty} \frac{k}{1+3k}$

b_k fails to converge to 0

$\lim_{k \to \infty} b_k = 1/3$

```
In[264]:= b[k_] := k/(1 + 3 k)
Limit[Abs[b[k]], k → Infinity]
(* Diverges by the divergence test. *)
```

Out[265]= $\frac{1}{3}$

2. Interval of convergence

For each power series, determine the interval of convergence. (Be sure to check endpoints, if there are any.)

whoops I forgot the $(-1)^{k+1}$, it switches the convergent & divergent endpoints.

1. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{2^{k-1} k}$

```
b[x_, k_] := x^k/(2^(k-1) k)
Limit[Abs[b[x, k+1]/b[x, k]], k → Infinity]
b[2, k]
(* Diverges, by comparison with the harmonic series. *)
b[-2, k]
(* Converges, by comparison with the alternating harmonic series. *)
```

$(-1)^{k+1}$

Out[272]= $\frac{\text{Abs}[x]}{2}$

Out[273]= $\frac{2}{k}$ $(-1)^{k+1}$ converges

Out[274]= $\frac{2\ (-1)^{k}}{k}$ diverges

$\boxed{\text{Interval: } (-2, 2]}$

2. $\sum_{k=0}^{\infty} (-1)^k \frac{(3x-7)^k}{k!}$

```
In[275]:= b[k_] := (-1)^k (3 x - 7)^k/(k!)
Limit[Abs[b[k+1]/b[k]], k → Infinity]
```

Out[276]= 0

$$\lim_{k\to\infty} \left| \frac{(-1)^{k+1} \frac{(3x-7)^{k+1}}{(k+1)!}}{(-1)^k \frac{(3x-7)^k}{k!}} \right| = \lim_{k\to\infty} \left| \frac{k!}{(k+1)!} (3x-7) \right| = |3x-7| \lim_{k\to\infty} \frac{1}{k+1} = 0$$