Section Summary: Arc Length

(I) Definitions

The **arc length formula**. If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

The **arc length function**. If smooth curve C has an equation $y = f(x)$, $a \leq x \leq b$, let $s(x)$ be the distance along C from the initial point $P_0(a, f(a))$ to the point $Q(x, f(x))$. Then

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2} dt$$

(note the change in the dummy variable of integration).

(II) Properties/Tricks/Hints/Etc.

Notice how the formulas change if integrating along the y axis.

(III) Summary

Archimedes got the whole ball started by computing the value of π by approximating a circle by a regular polygon, both inside and out. He thus squeezed the value of π down to between $3\frac{10}{71} < \pi < 3\frac{1}{7}$.

He thus approximated a smooth curve by a bunch of line segments, a process which we continue here. As the line segments get finer and finer, the approximation gets better and better, until, in the limit, the approximation becomes exact. We can imagine that we're using an approximation f^* to f, and hoping that the approximation of the arc length of f^* gets better and better as the line segments become shorter and shorter.

We do invoke the Mean Value Theorem at some point. You might want to remind yourself of how that works!