Overview of Sections 1.1-1.4 and 2.1

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Abstract

Your test will resemble the problems from your homework assignments, and problems from previous tests I've given. You will probably have 5 equally weighted questions or so (one every ten minutes!), one of which **may** involve several true/false questions (like the self-tests at the end of each chapter - answers are at the end of the book).

1 Section 1.1

We are introduced to statements, logical connectives, and wffs.

An implication is an argument, or theorem, which we may seek to prove. It is false if and only if the hypothesis (antecedent) is true while the conclusion (consequent) is false. The truth table for this logical connective is very important for understanding much of what follows!

Truth tables can prove tautologies (statements which are always true).

TautologyTest can prove tautologies of the form $P \to Q$, which it does by contradiction: assume both P and Q', and then break down each until all statement letters have truth values. If a statement letter is both true and false (a contradiction) then $P \wedge Q'$ is false, and the implication is true - a tautology.

2 Section 1.2

Propositional logic allows us to test arguments

$$P_1 \wedge P_2 \wedge \cdots \wedge P_n \rightarrow Q$$

to see if they're valid (tautologies).

Create a proof sequence using hypothesis or derivation rules (e.g. modus ponens). There are equivalence rules (such as DeMorgan's laws), and inference rules (e.g. modus tollens) which only operate in one direction.

The deduction method helps us prove implications: the antecedent joins the list of hypotheses, and we simply prove the consequent of the implication.

One seemingly difficult task is converting English arguments into wffs.

3 Section 1.3

We add a variable to statements to create predicate wffs. We then consider statements like "for all integers....", or "there is an integer such that....": that is, we quantify the predicate, using $\forall$ and $\exists$.

By introducing a variable we require a domain, called the domain of interpretation (non-empty).

Quantifiers have a scope, which indicates the part of a wff to which the quantifier applies.

Once again, translating English arguments into wffs is one of the tough challenges.

A few rules of thumb:

- $\forall$ tends to go with $\rightarrow$
- $\exists$ tends to go with $\wedge$

4 Section 1.4

We use predicate logic to prove predicate wffs, including new rules such as instantiation and generalization (as well as all the old familiar propositional logic rules).

Big Idea: strip off the quantifiers, use derivation rules on the wffs, and put quantifiers back on as necessary. Table 1.17 outlines limitations on stripping and putting.

A few rules of thumb:

- ei before ui.
- Don't use ug on $P(x)$ deduced from a hypothesis in which x is free, or by ei from another wff in which x is free:

1.$P(x)$	*hyp*
2.$(\forall x)P(x)$	incorrect ug

1.$(\forall x)(\exists y)Q(x,y)$	*hyp*
2.$(\exists y)Q(x,y)$	$1, ui$
3.$Q(x,a)$	$2, ei$
4.$(\forall x)Q(x,a)$	incorrect ug

5 Section 2.1

We look at a variety of proof techniques, including exhaustion, by contradiction, by contraposition, direct; and one "disproof" technique: counterexample.

Table 1: Summary of useful proof techniques, from Gersting, p. 91.

Proof Technique	**Approach to Prove** $P \to Q$	**Remarks**
Exhaustive Proof	Demonstrate $P \to Q$ for all cases.	Cases finite
Direct Proof	Assume P, deduce Q.	Standard approach
Contraposition	Assume Q', deduce P'.	Q' gives more ammo?
Contradiction	Assume $P \wedge Q'$, deduce contradiction.	