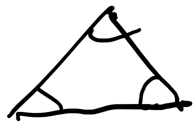
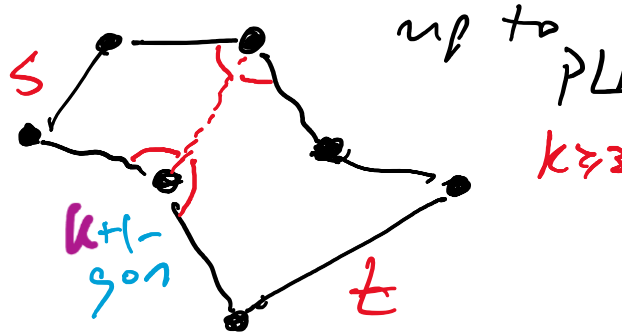
#72b Using 2^{nd} principle, prove that the sum of interior angles of an n-sided simple polygon is $(n-2)\,180^\circ$ for all $n \geq 3$.

* $n=3$ (base case)
(triangle).



Sum of interior angles is 180°
$= (3-2)\cdot 180^\circ$ ✓
$P(3)$ is true.

* Assume true for $P(r)$ up to $P(k)$:



Sum of interior angles of the s- & t-gons is the sum for the k+1-gon.

$s+t = k+1+2$
(the number of sides of the "gons")

The sum of the interior angles for the s- & t-gons follows the rule:

$(s-2)180^\circ + (t-2)180^\circ =$
$[\underbrace{s+t-2}\ -2]180^\circ =$
$[\ (k+1)\ -2\ ]180^\circ$ ✓

The sum of the interior angles of the k+1-gon follows the rule: $\therefore P(k+1)$

$\therefore P(n)\ \forall n \geq 3$ ✓ QED
by the 2^{nd} principle of induction.

Show that 2nd principle → well-ordering

Well-ordering: every non-empty collection of natural numbers has a smallest member.

Assume not: Let S be a set of natural numbers that has no smallest.

$P(n)$: $n \notin S$

* Show $P(1)$. Well if $1 \in S$, then it would be the smallest member! $\therefore P(1)$ ✓

* Assume $P(r)$ for all $1 \le r \le k$. Consider $P(k+1)$, which says $k+1 \notin S$. That's true, because if $k+1 \in S$, then it would be the smallest member - since $1, 2, \ldots, k$ are not in S by assumption. $\therefore P(k+1)$. ✓

$\therefore P(n)\ \forall n \in \mathbb{N}$, by the 2nd principle of induction.

But this is a contradiction: S was declared non-empty: i.e. $(\exists x)\ x \in S$. But that's false! $\therefore$ the principle of Well-Ordering is established.