#17

a. $(\exists x)[P(x) \wedge (\forall y)(T(y) \to F(x,y))]$

$(\exists y)[P(y) \wedge (\forall x)[T(x) \to F(y,x)]]$

rewritten reversing variables
No logical change!

b. $(\forall x)[P(x) \to (\exists y)[T(y) \wedge F(x,y)]]$

c. $[(\forall x)[P(x) \to (\forall y)[T(y) \to F(x,y)]]]'$

$\longleftrightarrow$

$(\exists x)[P(x) \to (\forall\ )[T(y) \to F(x,y)]]'$

Make you nervous?
Saved by the negation!

$\longleftrightarrow$ Implication

$(\exists x)[P(x)' \vee (\forall y)[T(y) \to F(x,y)]]'$

$\longleftrightarrow$ de Morgan

$(\exists x)[(P(x)')' \wedge ((\forall y)[T(y) \to F(x,y)])']$

dn

negation over universal

$$\longleftrightarrow (\exists x)\left[P(x) \wedge (\exists y)[T(y) \to F(x,y)]'\right]$$

$$\longleftrightarrow (\exists x)\left[P(x) \wedge (\exists y)[T(y)' \vee F(x,y)]'\right]$$

$$\longleftrightarrow (\exists x)\left[P(x) \wedge (\exists y)[(T(y)')' \wedge F(x,y)']\right]$$

$\downarrow$ dn

$$\longleftrightarrow (\exists x)\left[P(x) \wedge (\exists y)[T(y) \wedge F(x,y)']\right]$$

$$\longleftrightarrow (\exists x)(\exists y)\left[P(x) \wedge T(y) \wedge F(x,y)'\right]$$

My favorite way to think of this - an exists / exists!

#21 $(\forall x)\left[W(x) \wedge L(x) \to (\exists y)[J(y) \wedge A(x,y)]\right]$

Don't forget:
$\forall$ with $\to$
$\exists$ with $\wedge$

#22 $[(\exists x)(C(x) \wedge F(x))]'$

$\longleftrightarrow (\forall x)[C(x) \wedge F(x)]'$

$\longleftrightarrow (\forall x)[C(x)' \vee F(x)']$

$\longleftrightarrow (\forall x)[C(x) \rightarrow F(x)']$

$\longleftrightarrow (\forall x)[F(x) \rightarrow C(x)']$

All of these are legal answers! They're logically equivalent.

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b. $(\exists x)[P(x) \wedge (\forall y)[S(x,y) \rightarrow F(y)]]$

Not strong enough

c. ✗ $(\forall x)[C(x) \rightarrow (\forall y)(P(y) \rightarrow S(x,y))]$

$(\forall x)(\forall y)[S(x,y) \wedge P(y) \rightarrow C(x)]$

k. $(\forall x)[F(x) \to (\exists y)(C(y) \land S(x,y))]$