HW §3.1 #3, 17, 24

#3 $A(1) = 2$

$A(n) = \frac{1}{A(n-1)}$ $n \geq 2$

$A(2) = \frac{1}{A(1)} = \frac{1}{2}$

$A(3) = \frac{1}{A(2)} = \frac{1}{1/2} = 2$

$A(4) = \frac{1}{2}$

$A(5) = 2$

Closed form:

$$A(n) = \begin{cases} 2 & n \text{ odd} \\ \frac{1}{2} & n \text{ even} \end{cases}$$

Just for fun:

$$= \boxed{2^{(-1)^{n+1}}}$$

Cool!

Prove:

#17 $F(n+3) = 2F(n+1) + F(n) \quad n \geq 1$

Directly from the definition:

$$F(n+3) = F(n+2) + F(n+1) \quad \text{(by defn)}$$

$$= \big(F(n+1) + F(n)\big) + F(n+1)$$

$$= 2F(n+1) + F(n) \checkmark$$

This works from $\underline{n = 1}$

$\therefore$ Q.E.D.

#24. Prove:

$P(n)$: $F(n+6) = 4F(n+3) + F(n)$ for $n \geq 1$

Use the 2nd principle of induction.

This is a 6th order linear recurrence equation, so we might have to check it for up to six cases. In this example, however, we will need only 2 (as we will see):

$F_1 = 1$

$F_2 = 1$

$F_3 = 2$

$F_4 = 3$

$F_5 = 5$

$F_6 = 8$

$P(1)$: $F_7 = 13 = 4 \cdot 3 + 1 = 4 \cdot F_4 + F_1$

$P(2)$: $F_8 = 21 = 4 \cdot 5 + 1 = 4 \cdot F_5 + F_2$

Assume $P(r)$ for $k \geq r \geq 1$.

Consider $P(k+1)$, or specifically its LHS:

$$F((k+1)+6) = F(k+6) + F(k+5)$$

$$= \left(4F(k+3) + F(k)\right) + \left(4F(k+2) + F(k-1)\right)$$

(by assumption of $P(k)$ & $P(k-1)$)*

Here's where we see that we need* only two previous cases to carry on.

$$= 4\left[F(k+3) + F(k+2)\right] + F(k) + F(k-1)$$

$$= 4\left[F(k+4)\right] + F[k+1]$$

(by defn. of Fibonacci's)

$$= 4F[(k+1)+3] + F[(k+1)]$$

$\therefore P(k+1)$

$\therefore P(n)\ \forall\, n \geq 1$ by the 2^{nd} Principle of mathematical induction