

Hw §3, #1, 17, 24

#3  $A(1) = 2$

$$A(n) = \frac{1}{A(n-1)} \quad n \geq 2$$

$$A(2) = \frac{1}{A(1)} = \frac{1}{2}$$

$$A(3) = \frac{1}{A(2)} = \frac{1}{1/2} = 2$$

$$A(4) = \frac{1}{2}$$

$$A(5) = 2$$

Closed form:

$$A(n) = \begin{cases} 2 & n \text{ odd} \\ \frac{1}{2} & n \text{ even} \end{cases}$$

$$= \boxed{2^{(-1)^{n+1}}}$$

Cool!

Just  
for  
fun!

Prove:

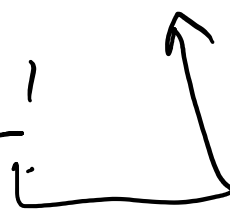
$$\#17 \quad F(n+3) = 2F(n+1) + F(n) \quad n \geq 1$$

Directly from the definition:

$$\begin{aligned} F(n+3) &= F(n+2) + F(n+1) \quad (\text{by defn}) \\ &= (F(n+1) + F(n)) + F(n+1) \\ &= 2F(n+1) + F(n) \quad \checkmark \end{aligned}$$

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This works from  $n=1$



$\therefore$  Q.E.D.

#24. Prove:

$P(n): F(n+6) = 4F(n+3) + F(n)$  for  $n \geq 1$   
Use the 2<sup>nd</sup> principle of induction.

This is a 6<sup>th</sup> order linear recurrence equation, so we might have to check it for up to six cases. In this example, however, we will need only 2

$$F_1 = 1$$

$$F_2 = 1$$

$$F_3 = 2$$

$$F_4 = 3$$

$$F_5 = 5$$

$$F_6 = 8$$

(as we will see):

$$P(1): F_7 = 13 = 4 \cdot 3 + 1 = 4 \cdot F_4 + F_1$$

$$P(2): F_8 = 21 = 4 \cdot 5 + 1 = 4 \cdot F_5 + F_2$$

Assume  $P(r)$  for  $k \geq r \geq 1$ .

Consider  $P(k+1)$ , & specifically its LHS:

$$F((k+1)+6) = F(k+6) + F(k+5)$$

$$= (4F(k+3) + F(k)) + (4(F(k+2) + F(k-1)))$$

Here's where we see that we need \* only two previous cases to carry on.  
(by assumption of  $P(k)$  &  $P(k-1)$ )\*

$$= 4[F(k+3) + F(k+2)] + F(k) + F(k+1)$$

$$= 4[F(k+4)] + F(k+1)$$

(by defn. of Fibonacci's)

$$= 4F[(k+1)+3] + F[(k+1)]$$

$$\therefore P(k+1)$$

$\therefore P(n)$   $\forall n \geq 1$  by the 2<sup>nd</sup>

Principle of mathematical induction





