§3.2, #6, 23, 24

#6. $S(1) = 1$

$S(n) = S(n-1) + (2n-1) \quad n \geq 2$

You can do this using the general closed form solution for a 1st order linear recurrence relation; or (easier) guess check & verify:

$$\left.\begin{aligned} S(2) &= 1 + (2\cdot 2 - 1) = 4 \\ S(3) &= 4 + (2\cdot 3 - 1) = 9 \\ S(4) &= 9 + (2\cdot 4 - 1) = 16 \end{aligned}\right\} \text{ squares?}$$

$P(n)$: $S(n) = n^2$

We've already checked $S(1)$.

Assume $P(k)$, & consider $P(k+1)$ (specifically the LHS:)

$S(k+1) = S(k) + (2(k+1) - 1)$

$= k^2 + 2k + 2 - 1$

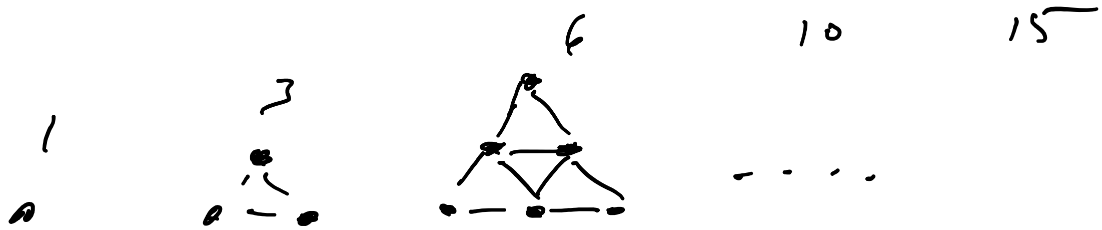
$= k^2 + 2k + 1 = (k+1)^2$ ✓

$\therefore P(k+1)$

$\therefore P(n)$ for all $n \in \mathbb{N}$.

\# 24 (is the recurrence relation for #6! So I'll do :))

\# 23



$S(1) = 1$

$$\boxed{S(n) = S(n-1) + n}$$

$S(2) = 3 = 1+2$

$S(3) = 6 = 1+2+3$

$S(4) = 10 = 1+2+3+4$

$\vdots$

$$\boxed{S(n) = \frac{n(n+1)}{2}} = \sum_{i=1}^{n} i$$

Gauss!

#41 Prove that the number of binary strings of length n with no two consecutive 0s is given by Fibonacci $F(n+2)$.

$n=1$: $S_1=\{0,1\}$ $\quad Card(S_1)^{*}=2=F(3)$

Card - is "cardinality" from section 4.1.

$n=2$: $S_2=\{01,10,11\}$ $\quad Card(S_2)=3=F(4)$

$n=3$: $S_3=\{010,110,011,101,111\}$ $\quad Card(S_3)=5=F(5)$

So far, so good! But I'm getting tired.... So let's see how this works....

S_n - strings of length n; those ending in 1 are created from all strings of length $n-1$, by adding a 1 to the end.

$S_n = Stringcat(S_{n-1}, "1") \cup^{*}$ those that end in 0.

$\cup$ - union - also from section 4.1

All those ending in 0 are created from strings of length $n-2$, by adding "10" to the end

So $S_n = \text{String cat}(S_{n-1}, \text{"1"}) + \text{String cat}(S_{n-2}, \text{"10"})$

So $\text{Card}(S_n) = \text{Card}(S_{n-1}) + \text{Card}(S_{n-2})$.

Since the number of binary strings satisfies the Fibonacci recurrence relation, but starts with F_3 & F_4, This is just the Fibonacci's shifted up two; so

$$\boxed{\text{Card}(S_n) = F_{n+2}}$$ ✓