§4.1 Homework ; p239 –

#6, 13, 39, 100

#6 a. $\{x \mid x \in \mathbb{N} \wedge x^2 - 5x + 6 = 0\}$

$(x-3)(x-2) = x^2 - 5x + 6 = 0$

$\rightarrow x = 3 \vee x = 2$

$\boxed{\{2, 3\}}$

b. $\{x \mid x \in \mathbb{R} \wedge x^2 = 7\} =$

$\boxed{\{-\sqrt{7}, \sqrt{7}\}}$

c. $\{x \mid x \in \mathbb{N} \wedge x^2 - 2x - 8 = 0\}$

$= \{x \mid x \in \mathbb{N} \wedge (x-4)(x+2) = 0\}$

$= \boxed{\{4\}}$

#13 Let $R = \{1, 3, \pi, 4.1, 9, 10\}$

$S = \{\{1\}, 3, 9, 10\}$

$T = \{1, 3, \pi\}$

$U = \{\{1, 3, \pi\}, 1\}$

$\{1\} \notin R$ a. $S \subseteq R$ ✗

b. $1 \in R$ ✓

$\{1\} \in S$ c. $1 \in S$ ✗

$\{1\} \subseteq U$ d. $1 \subseteq U$ ✗

e. $\{1\} \subseteq T$ ✓

f. $\{1\} \subseteq S$ ✗

g. $T \subset R$ ✓

$\{\{1\}\} \subseteq S$

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#39 a. $x \circ y = x + 1$ $S = \mathbb{N}$

Perfectly good - just happens to ignore y....

b. $x \circ y = x + y - 1$ $S \in \mathbb{N}$ ✓

Author's wrong on this:

$0 \notin \mathbb{N}$. I'm going to see if you've been listening to me....

c. $x \circ y = \begin{cases} x-1 & x \text{ odd} \\ x & x \text{ even} \end{cases}$

No: $1 \circ 3 = 0 \notin \mathbb{N}$

fails closure.

d. $x^{\#} = \ln(x) \quad x \in \mathbb{R}$.

No: $\ln(0)$ isn't defined

$\ln(-1)$ " "

etc.

Not well-defined.

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#100 Prove that the set of all finite length binary strings is denumerable.

We need to show that the natural numbers "smother" these strings: that is, that there are plenty of natural numbers for each finite binary string.

In terms of Hotel $\infty$, every string gets its own room.

Consider a finite length binary string, e.g.

$$01011.$$

Send it to room

$$\underline{1}01011_2 =$$

$$\underbrace{2^5} + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2 + 1$$

$$= 43_{10}$$

Distinct binary strings clearly go to different rooms; & every binary string has a room.

$\therefore$ Card (binary strings)

$\leq \underbrace{\text{Card}(\mathbb{N})}_{\text{smallest infinity}}$.

So the set of all binary strings (of finite length)

is denumerable (countable - infinite, but of a rather small size... :) ).