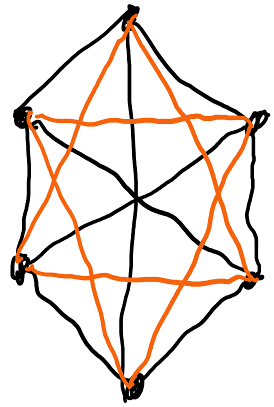
§6.1 HW #5, 28, 44 p438–
(every graph has 6 nodes!)

#5 Draw K_6:



I didn't assign this one, but mistook it for 28!

#29. If a simple, connected, planar graph has 6 nodes, all of degree 3, into how many regions does it divide the plane?

Interesting problem, anyway :)

Presumably we can count edges!

Since it's planar, $r-a+n=2$:

$$r - a + n = 2$$

$$\therefore r = 2 + a - n$$

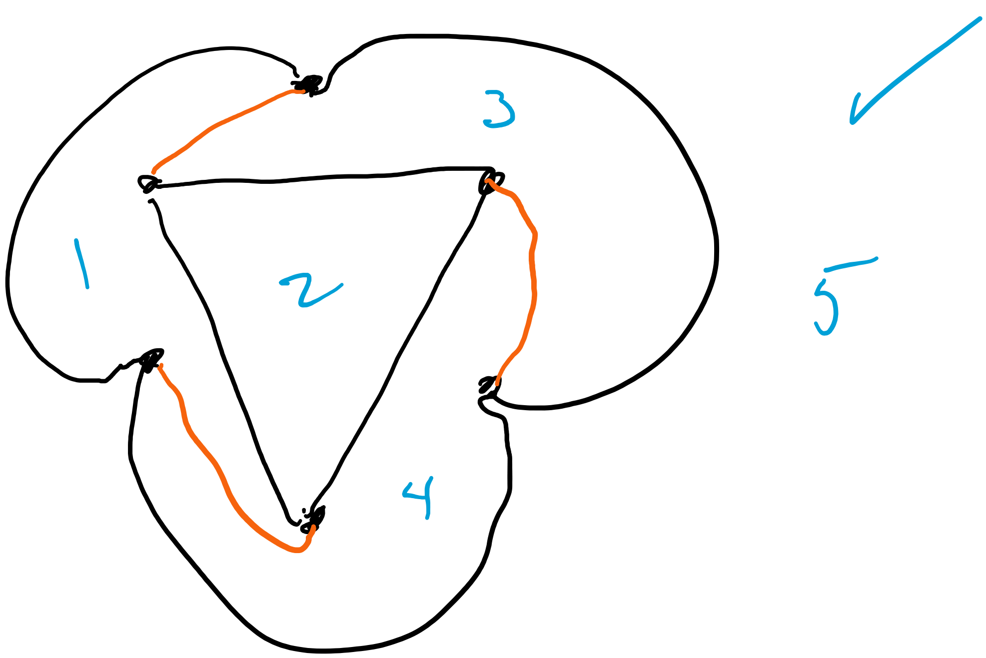
#28 below...

We know that $n = 6$; each node is of degree 3 $\rightarrow$ total degree $= 18$ $(= 6 \times 3)$; this double counts the arcs (each arc contributes 2 to the total degree).

Hence $a = 18/2 = 9$.

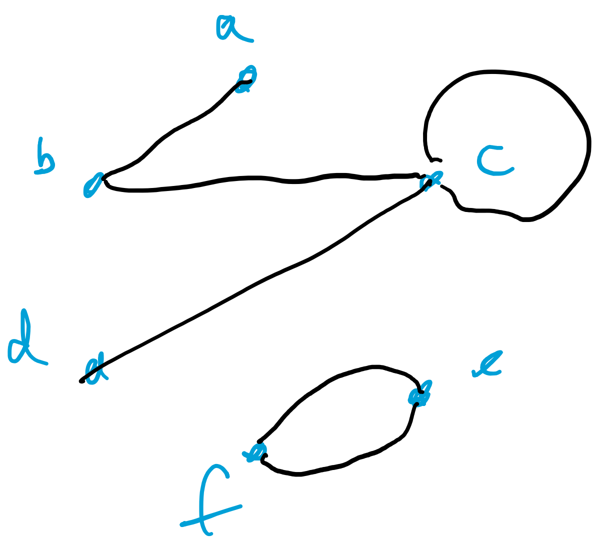
$$\therefore r = 2 + 9 - 6$$
$$= 5$$

Let's just check on an example:



#44

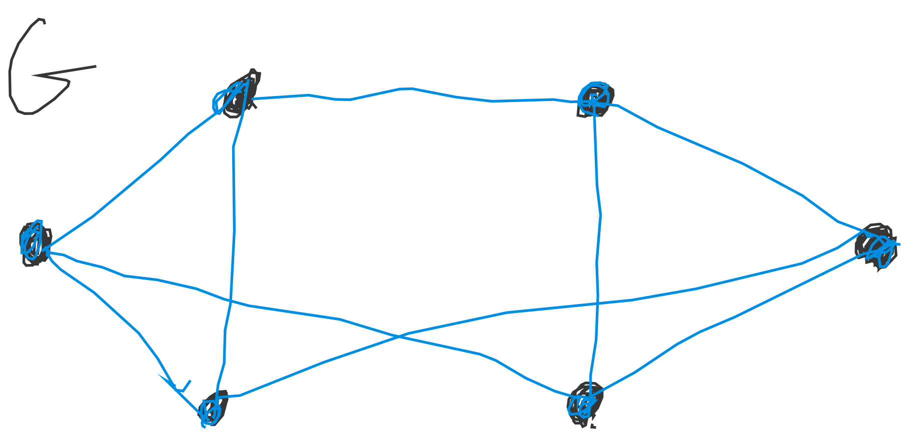
$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & 0 & 1 & 0 & 0 & 0 & 0 \\
b & 1 & 0 & 1 & 0 & 0 & 0 \\
c & 0 & 1 & 1 & 1 & 0 & 0 \\
d & 0 & 0 & 1 & 0 & 0 & 0 \\
e & 0 & 0 & 0 & 0 & 0 & 2 \\
f & 0 & 0 & 0 & 0 & 2 & 0
\end{array}
$$



#28

6 nodes

9 arcs

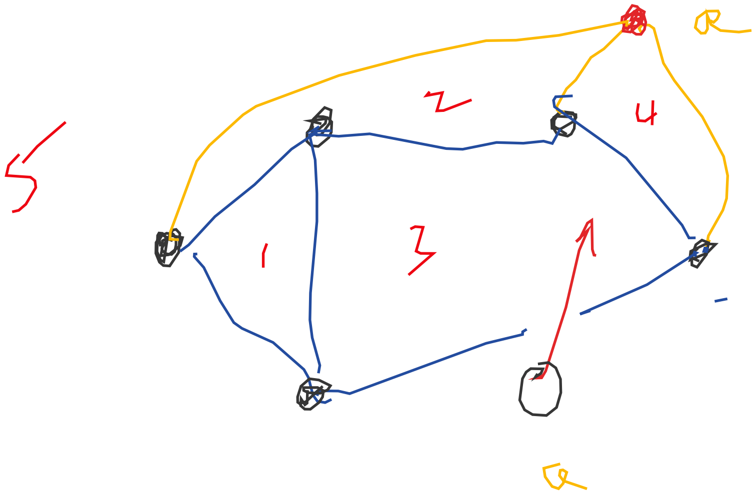


The easy proof that this graph

is planar is that a non-planar graph must contain a subgraph isomorphic to K_5 or K_{33}* — the complete graph on 5 vertices or the bipartite complete graph "three houses, three utilities".

Our graph G doesn't have any degree 4 nodes (which K_5 requires) & it has a cycle of order 3 (which no node of $K_{3,3}$ has).

* Kuratowski's Theorem.



Check:

$\boxed{5 - 9 + 6 = 2 \checkmark}$

Euler's formula.

Just move node a to recenter it, & you see that it was planar all along....