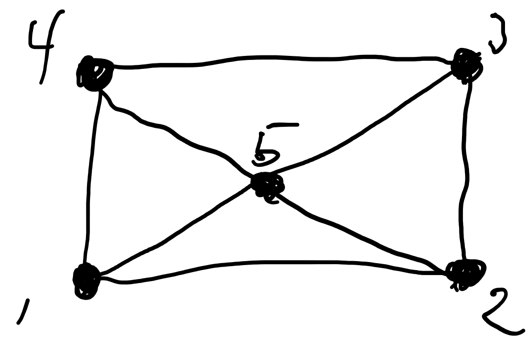
§7.2 Homework

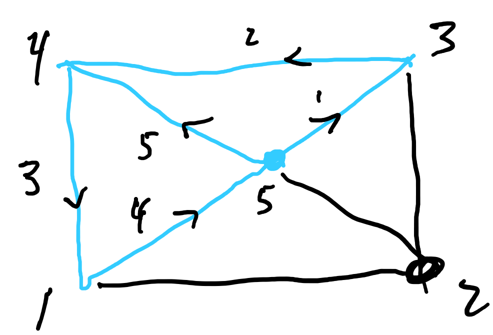
#1, 16, 31.

#1. 

Prove the conjecture "It is not possible to trace all the lines in the figure without lifting your pencil or without retracing any lines."

Now it's easy: four nodes of odd degree imply that there exists no Euler path.

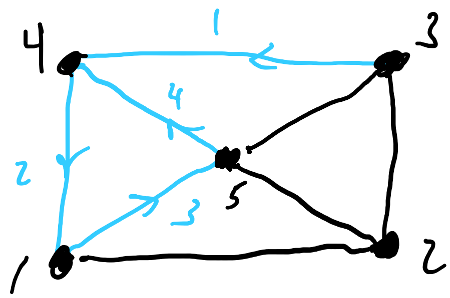
It's an interesting question to ask "what's the worst you can do? Start at 5:



Is that worst?

No....

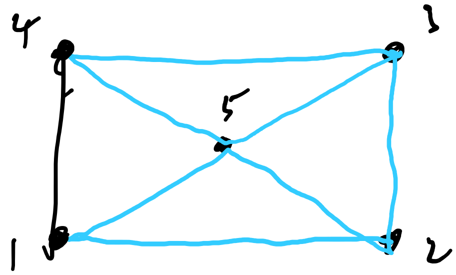
Start at 3:



Can you do any worse? :)

What's the best? All but one.... (of course - because if we blow up one bridge between two

e.g



nodes of degree 3 they become even, & we have only two odd degree nodes).

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16. Eight nodes, Darn... :)

No loops (0 diagonal);

Undirected (so symmetric)

	1	2	3	4	5	6	7	8
1	0	1	0	1	0	0	0	0
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	1	0	1	0	0	0	0	1
5	0	0	0	0	0	1	0	1
6	0	1	0	0	1	0	1	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	1	0	1	0

→ Total = 3

$i = 5$

break.

Complete Graph:

#31 a. n nodes; start from 1 (without loss of generality).

$(n-1)$ choices for departure;
$(n-1)$ choices at each additional step.

$\boxed{(n-1)^n}$ (path of length n, so n choices).

b. $(n-1)$ choices for the 1st; but only $(n-2)$ for successive because we can't repeat one arc.

$$(n-1)(n-2)^{n-1}$$

c. $(n-1)\underbrace{(n-2)}_{\text{we lose one more option at each step....}}\cdots(n-n) = (n-1)!$

d. If $n=15$,

$(n-1)! = 14! = 87178.291200$ .000001 s/operation

So about 87178 seconds; a little over a day ($60*60*24 = 86400$ sec).

So an exhaustive approach is very slow (& growing factorially, which means extraordinarily rapid growth in the time — as $n!$).