§8.1 #10acd & 15

#10a $(x+y)+(y\cdot x')=x+y$ (see bottom)

Start w/ the left-hand side, & hope that we can get to the right.

a. $(x+y)+(y\cdot x') =$ ("weird distributivity")

$((x+y)+y)\cdot((x+y)+x') =$ (associativity & commutativity)

$(x+(y+y))\cdot((y+x)+x') =$ (idempotence & associativity)

$(x+y)\cdot(y+(x+x')) =$ (complements)

$(x+y)\cdot(y+1) =$ (practice 3)

$(x+y)\cdot 1 = (x+y)$ ✓

(identity)

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c. $(y'\cdot x)+x+(y+x)\cdot y' = x+y'\cdot x$

= distributivity

$y'\cdot x + x + (y\cdot y' + x\cdot y') =$ (complements)

$y'\cdot x + x + (0 + x\cdot y') =$ (identity)

$y'\cdot x + x + x\cdot y' =$ (commutativity)

$y' \cdot x + x + y' \cdot x$ = (commutativity)

$y' \cdot x + y' \cdot x + x$ = (idempotence)

$y' \cdot x + x = x + y' \cdot x$ ✓

commutativity

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d. $(x + y') \cdot z = [(x' + z') \cdot (y + z')]'$

Start right, & try to get left:

demorgan: RHS $= (x' + z')' + (y + z')'$

" : $= x'' \cdot z'' + y' \cdot z''$

double negation: $= x \cdot z + y' \cdot z$

distributivity $= (x + y') \cdot z$

#15 Define $x \oplus y = x \cdot y' + y \cdot x'$

Prove:

a. $x \oplus y = y \oplus x$

LHS $= x \cdot y' + y \cdot x' = y \cdot x' + x \cdot y'$ (commutativity)

$=$ RHS ✓

b. $x \oplus x = 0$

complements, identity

$LHS = x \cdot x' + x \cdot x' = 0 + 0 = 0$

$= RHS$ ✓

c. $0 \oplus x = x$

practice 3 dual, complements

$LHS = 0 \cdot x' + x \cdot 0' = 0 + x \cdot 1$

$= x$ ✓ (identity)

d. $1 \oplus x = x'$

identity, practice 3 dual

$LHS = 1 \cdot x' + x \cdot 1' = x' + x \cdot 0 = x' + 0$

$= x'$ ✓ (identity)

I like Ngo's solution to 10 better than mine:

$$(x+y) + (y \cdot x') =$$

$$x + (y + (y \cdot x')) =$$

$$x + (y \cdot 1 + y \cdot x') =$$

$$x + y \cdot (1 + x') =$$

$$x + y \cdot 1 =$$

$$x + y$$