#4. a. $(\forall x)[D(x) \wedge S(x) \rightarrow R(x)] \rightarrow (\forall x)[D(x) \wedge S(x) \rightarrow D(x) \wedge R(x)]$

1.	$(\forall x)[D(x) \wedge S(x) \rightarrow R(x)]$	hyp
2.	$D(x) \wedge S(x) \rightarrow R(x)$	1, ui
3.	$D(x) \wedge S(x)$	temp hyp
4	$D(x)$	3, simp
5.	$R(x)$	2, 3, MP
6.	$D(x) \wedge R(x)$	4, 5, conj
7.	$D(x) \wedge S(x) \rightarrow D(x) \wedge R(x)$	temp hyp discharged
8.	$(\forall x)[D(x) \wedge S(x) \rightarrow D(x) \wedge R(x)]$	7, ug

Good work

b. $(\forall x)[S(x) \vee R(x)] \wedge (\exists x) R(x) \rightarrow (\exists x) S(x)$

Domain: Positive Integers

$F: S(x) - x < 1$; $R(x) - x \geq 1$

Good!

* The only interpretation that the above statement is going to be invalid is if the antecedent is True and the consequent is False

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#1 a) ii

b)

i) $C \wedge (S' \vee R')$

ii) If it is not cloudy and not rainy then it will be sunny

iii) $C' \wedge R' \rightarrow S$

$(S' \rightarrow (C' \wedge R'))'$

$S' \rightarrow C \vee R$

→ if it not sunny therefore it is cloudy or rainy.

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#2 a) $(I \rightarrow Q) \wedge (I \vee D) \wedge Q' \rightarrow D$

b)

1)	$I \rightarrow Q$	hyp
2)	$I \vee D$	hyp
3)	Q'	hyp
4)	I'	1, 3 mt
5)	$I' \rightarrow D$	2, imp
6)	D	4, 5 imp

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(3)

a. ∀

i. $(\forall x)[\, H(x) \rightarrow M(x)\,]$ ✓

ii $(\exists x)[\, H(x) \wedge L(x)\,]$ ✓

iii $(\forall x)\,[\, H(x) \rightarrow (\exists y)\,[\, R(y) \wedge C(x,y)]\,]$ ✓

iv. $(\forall x)\,[\, H(x) \rightarrow (\exists y)\,[\, R(y) \wedge C(x,y)'\,]\,]$ ✓

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b.

i. There exists some hyenas that are meat-eaters and laugh and chase all rodents. ✓

ii For all rodents, they will be chased by all hyenas ✓

6 D 4,5 M2

3ai $(\forall x)[H(x) \rightarrow M(x)]$ ✓

ii $(\exists x)[H(x) \wedge L(x)]$ ✓

iii $(\forall x)[H(x) \rightarrow (\exists y)(R(y) \wedge C(x,y))]$ ✓

iv $(\forall y)[(R(y) \wedge C(x,y)) \rightarrow H(x)']$ ✓ ⇐⇒ no hyena chases a rodent. -.5

9.5

3bi Some meat-eating hyenas laugh and chase all rodents.

ii All rodents are chased by all hyenas. ✓

4b invalid
domain: natural numbers
$S(x)$: $x<0$
$R(x)$: $x>0$
"all x are either less than 0 or greater than 0. there are some natural numbers greater than 0. therefore there are some natural numbers less than 0." this is incorrect as we know all natural numbers are greater than 0.

5a We can prove this with two separate proofs: "if x is odd, then $3x+5$ is even" and "if $3x+5$ is even, then x is odd." We will start with "if x is odd, then $3x+5$ is even." Let $x=2a+1$ where $a \in \mathbb{R}$. So, $3x+5 = 3(2a+1)+5 = 6a+8 = 2(3a+4) = 2c$ where $c = 3a+4$. Since $3x+5$ is in the form $2c$, it is even. Now looking at the proof "if $3x+5$ is even, then x is odd," we can solve this by contraposition. Suppose x is even, so $x=2a$ where $a \in \mathbb{R}$. Then, $3x+5 = 3(2a)+5 = 6a+5 = 2(3a+2)+1$ $= 2c+1$ where $c = 3a+2$. Since $3x+5$ is in the form $2c+1$, it is odd. This proves by contraposition that if $3x+5$ is even, then x is odd. These two proofs show equivalence of the two statements.

You've assumed two things about x - that it's positive and that it's odd!

Nice work!

-5

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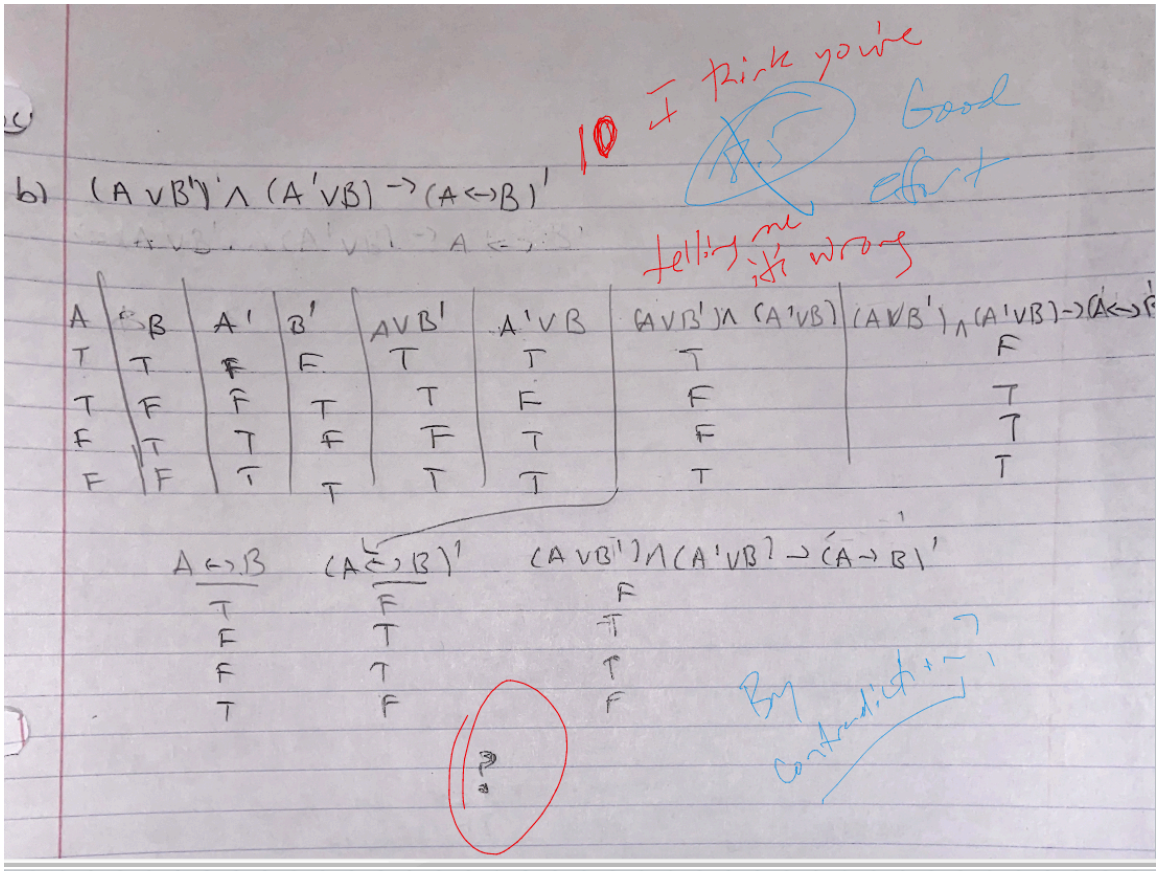
9.5



b) $(A \vee B')' \wedge (A' \vee B) \rightarrow (A \leftrightarrow B)'$

10 I think you're telling me it's wrong

Good effort

A	B	A'	B'	A∨B'	A'∨B	(A∨B')∧(A'∨B)	(A∨B')∧(A'∨B)→(A↔B)'
T	T	F	F	T	T	T	F
T	F	F	T	T	F	F	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	T

A↔B	(A↔B)'
T	F
F	T
F	T
T	F

(A∨B')∧(A'∨B) → (A→B)'

F
T
T
F

By contradiction

5b.
1. $A \vee B'$ hyp
2. $A' \vee B$ hyp
3. $(A \leftrightarrow B)''$ hyp
4. $A \rightarrow B$ 2, imp
5. $B' \vee A$ 1, comm
6. $B \rightarrow A$ 5, imp
7. $(A \rightarrow B) \wedge (B \rightarrow A)$ 4, 6, con
8. $(A \leftrightarrow B)$ 7, equ
9. $A \leftrightarrow B$ 3, den

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You're telling me nicely that I made a mistake...!

:)