

Exam 3 key

- A.) i) ^{1 2 3 4 5 6} Pre-order → ~~n k c b l m t p r o u~~ ✓
 ii) In-order → b c k l m n o p r t u ✓
 3.) iii) Post-order → b, c, m, l, k, o, r, p, u, t, n ✓

2.) Mistake is in preorder, it should be n, k, c, b, l, m, t, p, o, r, u ✓

4.) the in-order traversal is in alphabetical order. ✓

B. You want to store 374 distinct names in a binary search tree (BST): ^{d =} 373 - submit ordered

1. (3 pts) What's the maximum depth possible, and how is such a BST created? ^{order list, then re-}
^{d = 9} 2. (4 pts) What's the minimum depth possible, and how is such a BST created? ^{cursively submit middle}
 3. (3 pts) Assume we create a minimum-depth BST. What's the greatest number of comparisons possible when conducting a binary tree search, in the worst-case? ^{Smallest d /}

$2^d - 1 > 374$; $d = 9$ → 10 comparisons in worst case

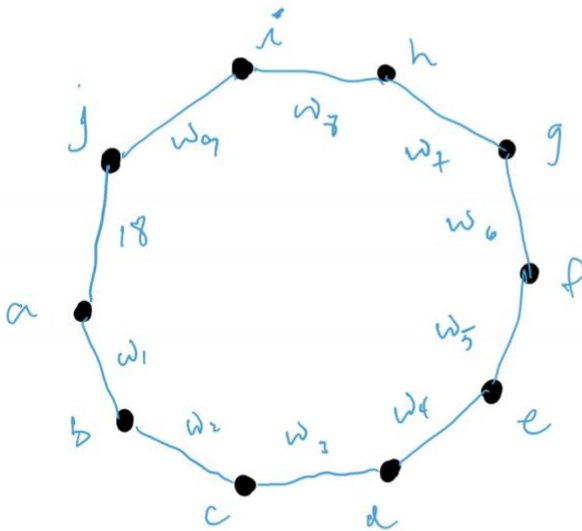
- c.) 1. A is planar, by looking at the degree of the nodes. No way to make
 2. B is not planar b/c it contains $K_{3,3}$ / K_5 or $K_{3,3}$

D) 1) This will likely be best plotted on an adjacency matrix because there are (relatively) few countries, and most countries have a lot of diplomatic relations. ✓

2) This will be best plotted as an adjacency list because there are thousands of cities with few or no air routes.

- E) 1) n, k, c, b, l, m, t, p, o, r, u this is preorder traversal ✓
 2) n, k, t, c, l, p, u, b, m, o, r ✓

F. Given this positively weighted graph:



- note: 8, not 9
- $\sum_{i=1}^8 w_i < 18$
 - Distance is the sum of the weights between a + the nodes,

Suppose we want to find the shortest path from node a to node j, and we use Dijkstra's algorithm. The distance between a and j is 18. If j is the last node settled,

- what do we know about the distances w_i ?
- what is the distance between a and each node?

$$i=1 \dots 4 \quad D(e) = \sum_{i=1}^4 w$$

$$D(j) = \min\left(18, \sum_{i=1}^9 w_i\right), \text{ in particular.}$$

G
1: the number of distinctly different friendships is the number of arcs in the completed graph. The friendship between A and B is the same as the friendship between B and A.

4 nodes, completed graph \rightarrow each node has 3 edges \Rightarrow 12 edges

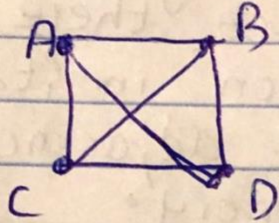
Each edge represents 1 arc \rightarrow 6 arcs

\Rightarrow Total of 6 friendships

2: The number of different sets of friendship is power set of the set of all friendships, which has 6 cardinals

\Rightarrow There are 2^6 cardinals in the power set, or 2⁶ different configurations of friendship between 4 people.

G) Soln



1) Six Friendship are possible betⁿ them. ✓

2) We can calculate distinct different Facebooks by using power set.
 $2^6 = 64$ possible Facebooks. ✓

H.

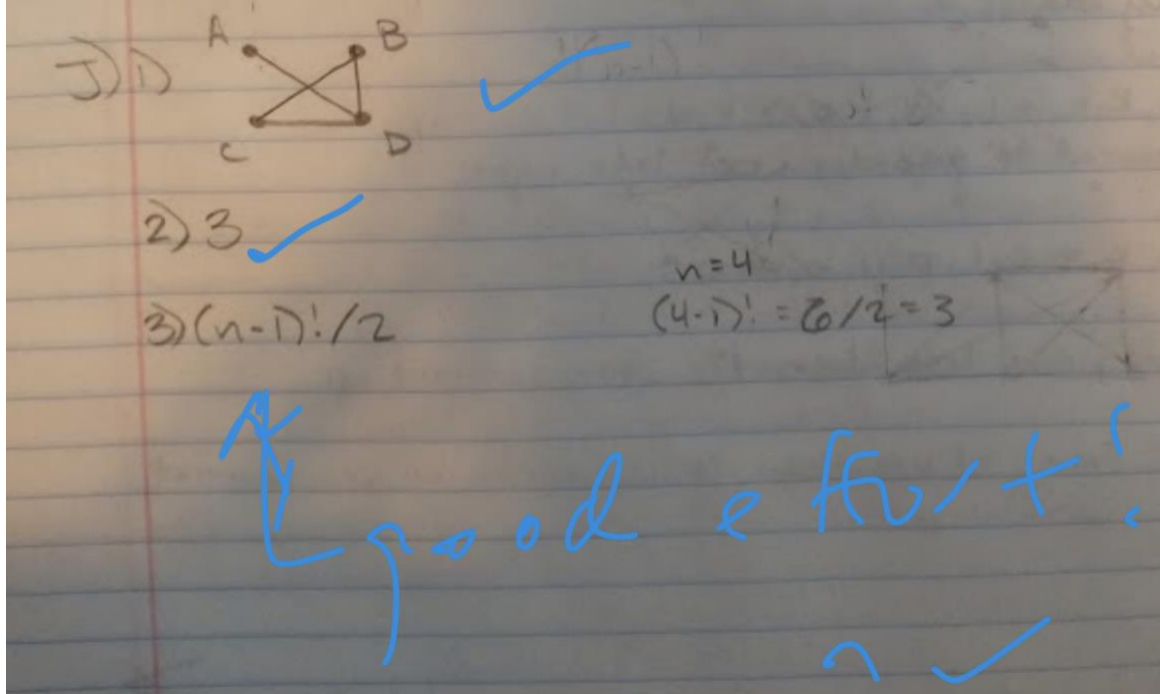
1. 2! Hamiltonian Circuits ✓

2. No because all the nodes would have an odd degree. Euler Path must have 0 nodes or 2 nodes with odd degrees. ✓

I) 1) $\{ \}$ can't be associated with a unique number ✓

2) ~~0~~ $2^0 - 1$ (for the empty set) different numbers ✓

3) All the integers with distinct digits in decreasing order from 0 to 9876543210.



Part 3: these graphs are complete graphs of order $(n-1)$ with a vertex of degree 1 tacked on. Sophia's is not the correct answer, but it's my bad: I didn't cover this example this semester, so you probably wouldn't have known or seen the formula n^{n-2} for a complete graph of n vertices). So what I looked for especially was a good guess, and a check that a proposed formula worked for the case of 4 vertices (the case we had here). On that count, Sophia did great!

