MAT385 Test 1 (Spring 2021): 1.1-1.4; 2-1

Name:

Directions: Problems are worth 10 points each. Show your work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answers to each problem (e.g., put a box around them); and clearly separate solutions to each problem from other problems. **Good luck!**

Problem 1: Propositional logic:

  a. (2.5 pts) Consider the compound wff *If the carton is sealed then the raisins are fresh.*

    Several potential forms of negation are given below for this sentence. Identify those which are correct (if any):

      i. If the carton is not sealed then the raisins are not fresh.

      ii. The carton is sealed and the raisins are not fresh.

      iii. Either the carton is sealed or the raisins are not fresh.

  b. Using the statement letters C (cloudy), S (sunny), R (rainy)

      i. (2.5 pts) Translate the compound wff "Yesterday was cloudy, but it was neither sunny nor rainy." into symbolic notation.

      ii. (2.5 pts) Translate the symbolic wff $C' \wedge R' \rightarrow S$ into English.

      iii. (2.5 pts) Write the contrapositive of $C' \wedge R' \rightarrow S$, and translate it into a reasonable English sentence.

Problem 2: If the fan is Installed properly, it runs Quietly; either the fan is Installed properly or it is Defective. However the fan does not run Quietly. Therefore it is Defective. (Use I – Installed properly, Q – runs Quietly, D – is Defective)

  a. (5 pts) Write the argument using propositional wffs. [Be careful: if you get this part wrong, then the next part will also likely be wrong!]

  b. (5 pts) Then prove it using propositional logic (but without the use of disjunctive syllogism).

Problem 3: Consider the following in the domain which is, as our author likes to say, "the whole world":

- $H(x)$: x is a hyena.

- $L(x)$: x laughs.

- $M(x)$: x is a meat-eater.

- $R(x)$: x is a rodent.

- $C(x, y)$: x chases y.

a. (6 pts) Using the predicate symbols above and appropriate quantifiers, write each English language statement as a predicate wff.

   i. All hyenas are meat-eaters.

   ii. Some hyenas laugh.

   iii. All hyenas chase some rodents.

   iv. No hyenas chase all rodents.

b. (4 pts) Give English language translations of the following wffs

   i. $(\exists x)\,[H(x) \wedge M(x) \wedge L(x) \wedge (\forall y)\,(R(y) \rightarrow C(x, y))]$

   ii. $(\forall x)\,[R(x) \rightarrow (\forall y)(H(y) \rightarrow C(y, x))]$

Problem 4: Predicate logic

a. (5 pts) For one of our homework exercises, most of you stated the answer one way, but I stated it another: many of you wrote
$$(\forall x)\,[D(x) \wedge S(x) \rightarrow D(x) \wedge R(x)]$$
I wrote
$$(\forall x)\,[D(x) \wedge S(x) \rightarrow R(x)]$$
Prove that my argument is just as strong, by showing that
$$(\forall x)\,[D(x) \wedge S(x) \rightarrow R(x)] \rightarrow (\forall x)\,[D(x) \wedge S(x) \rightarrow D(x) \wedge R(x)]$$
(Hint: temporary hypothesis works nicely.)

b. (5 pts) Prove valid or disprove using an appropriate interpretation:

$$(\forall x)\,[S(x) \vee R(x)] \wedge (\exists x)R(x) \rightarrow (\exists x)S(x)$$

Problem 5: : Do one (**your choice**) of the following two problems:

a. Prove or provide a counterexample (make no **assumptions** about x!):
x is an odd integer if and only if $3x + 5$ is an even integer.

b. Prove by contradiction:

$$(A \vee B') \wedge (A' \vee B) \rightarrow (A \leftrightarrow B)'$$