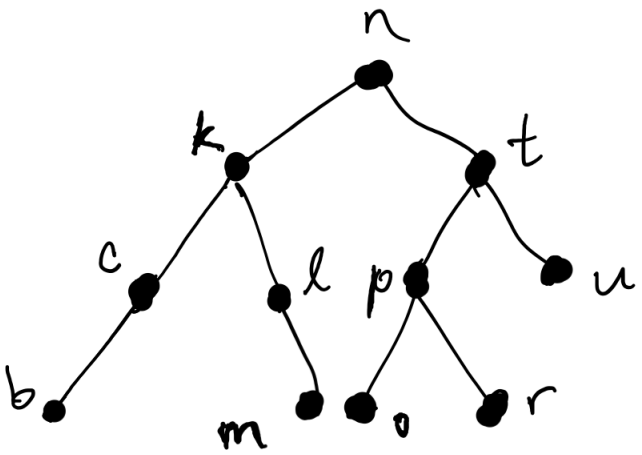
MAT385 Test 3 (Spring, 2021): 4.1, 6.1, 6.2, 6.3, 7.2, 7.3, 7.4

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck! You should skip one problem, of your choice. Write SKIP on it.**

A. Consider this tree:



A student has already written out two of three traversals (in-order, pre-order, post-order):

- i. n k c b l m t p r o u
- ii. b c k l m n o p r t u
- iii. ???

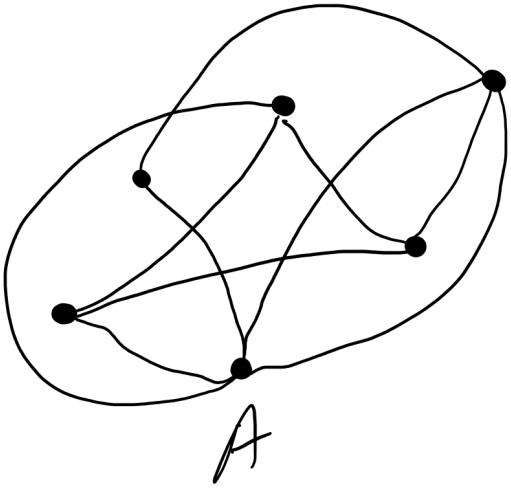
You are to

1. name which two traversals are already provided above,
2. fix a small error in one of them,
3. write the third traversal, and
4. comment on what is striking about the in-order traversal.

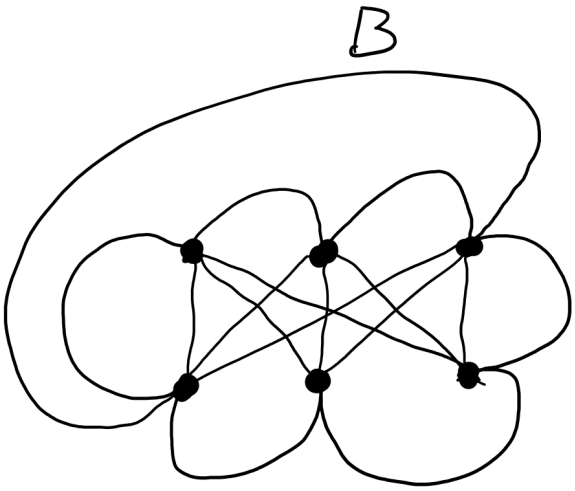
B. You want to store 374 distinct names in a binary search tree (BST):

1. (3 pts) What's the maximum depth possible, and how is such a BST created?
2. (4 pts) What's the minimum depth possible, and how is such a BST created?
3. (3 pts) Assume we create a minimum-depth BST. What's the greatest number of comparisons possible when conducting a binary tree search, in the worst-case?

C. You can tell at a glance that one of these graphs is planar, while the other isn't:



A



B

1. Which graph is planar, and how do you know by looking at it?
2. Which graph is not planar, and how do you know by looking at it?

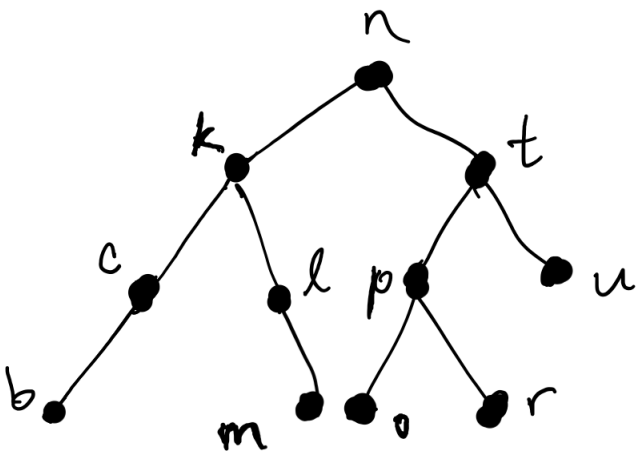
(You may not make reference to the other graph in your answer – e.g., you can't say, for example, that "I know that graph C is planar because graph D isn't.")

D. Consider these two graphs:

1. Graph whose nodes are countries of the world; edges represent existence of diplomatic relations between nodes.
2. Graph whose nodes are all cities of 20,000 residents or more in America; edges represent existence of a direct commercial air route between nodes.

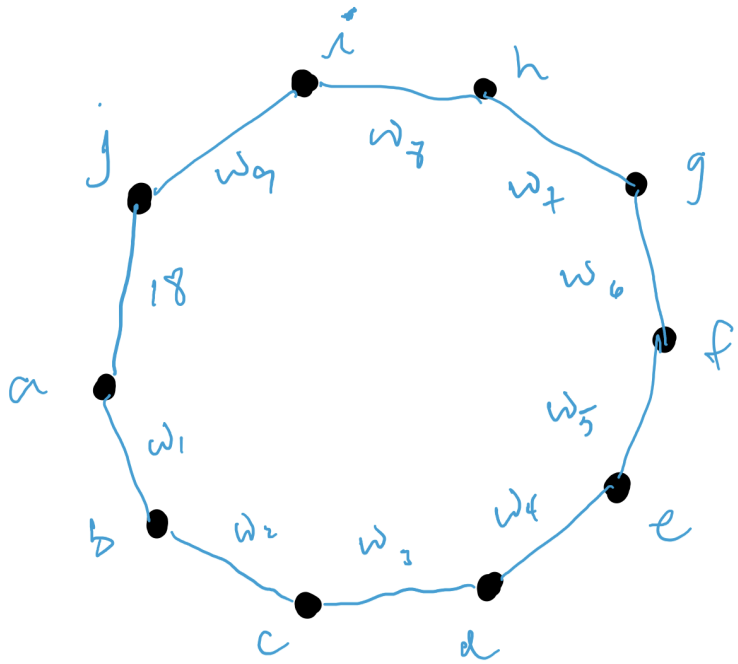
They would be very difficult to represent visually, but it's easy to represent them in a computer. Decide whether you would use an adjacency matrix or an adjacency list to represent them, and explain **why** (0 points without an explanation).

E. We can apply graph algorithms to trees: consider the tree



1. Perform a depth-first graph-traversal on the tree starting from root node n (and relate it to a well-known tree-traversal algorithm).
2. Perform a breadth-first graph-traversal on the tree starting from root node n.

F. Given this positively weighted graph:



Suppose we want to find the shortest path from node a to node j, and we use Dijkstra's algorithm. The distance between a and j is 18. If j is the last node settled,

1. what do we know about the distances w_i?
2. what is the distance between a and each node?

G. A Facebook graph is a simple graph with n nodes (distinct individuals), and a list of edges (friendships, which may or may not exist, depending on the individuals) between the nodes.

Assume a Facebook of 4 nodes (distinct people: A, B, C, D), and a number of edges (friendships) to be determined.

1. How many distinctly different friendships are possible? (Hint: How many edges in a complete graph?)
2. How many distinctly different Facebooks are possible (that is, different configurations of friendships between the four individuals)? (Hint: power sets!)

H. Consider a simple, complete graph with 22 nodes.

1. Suppose that the edges of this graph are positively weighted. To solve the traveling salesman problem, assume a fixed starting point at node 1 and generate all possible Hamiltonian circuits of length 22. How many different Hamilton circuits are there?
2. Is there an Euler path on this graph? (You must explain! 0 points for yes/no alone....)

I. Consider the set of ten digits $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. A subset of S such as $A = \{0, 3, 7\}$ can be considered a unique number, where the digits are ordered from largest to smallest: so $A \to 730$.

1. (2 pts) One subset can't be associated with a unique number. Which one?
2. (4 pts) How many distinctly different numbers can be generated in this way?
3. (4 pts) Characterize these numbers in words.

J. Spanning trees:

1. (3 pts) Draw a simple graph with four nodes A, B, C, and D, of degrees 1, 2, 2, and 3, respectively.
2. (4 pts) How many distinctly different spanning trees are there for this tree?
3. (3 pts) Generalize: suppose a simple graph with n nodes has nodes of degrees 1, $n-1$, and all the rest are of degree $n-2$. How many distinctly different spanning trees are there for this tree?