

§1.4 #15, 18, 42

#15 Prove valid:

$$(\forall x)(P(x)) \wedge (\exists x)(P(x))' \rightarrow (\exists x)Q(x)$$

1, $(\forall x)P(x)$	hyp	1, $P(x)$	1, u.i.
2, $(\exists x)(P(x))'$	hyp	2, $Q(x)$	6, 7, imp
3, $(P(x))'$	2 ei	3, $(\exists x)Q(x)$	8, e.g.
4, $P(x)' \vee Q(x)$	3, add		
5, $(P(x))' \rightarrow Q(x)$	4, imp		
6, $P(x) \rightarrow Q(x)$	5, dn		

#18, p. 71 Prove valid or give an interp. in which it is false.

$$\begin{aligned} * & (\exists x)[R(x) \vee S(x)] \rightarrow (\exists x)R(x) \vee (\exists x)S(x) \\ & \iff (\exists x)[R(x) \vee S(x)] \rightarrow [((\exists x)R(x))' \rightarrow (\exists x)S(x)] \text{ (imp)} \\ & \iff (\exists x)[R(x) \vee S(x)] \wedge ((\exists x)R(x))' \rightarrow (\exists x)S(x) \text{ (dn)} \end{aligned}$$

$$(\exists x) [R(x) \vee S(x)] \wedge ((\exists x) R(x))' \rightarrow (\exists x) S(x)$$

$$1. (\exists x) [R(x) \vee S(x)] \quad \text{hyp}$$

$$2. ((\exists x) R(x))' \quad \text{hyp}$$

$$3. (\forall x) R(x)' \quad \text{d, neg of } (\exists x)$$

$$4. R(x) \vee S(x) \quad \text{l, e.i.}$$

$$5. R(x)' \quad \text{3, u.i.}$$

$$6. S(x) \quad \text{4, 5 ds}$$

$$7. (\exists x) S(x) \quad \text{6, e.s.}$$

#42 Prove the argument valid

Every farmer owns a cow. No dentist owns a cow. Therefore no dentist is a farmer.

$F(x)$, $C(x)$, $D(x)$, $O(x,y)$ - x owns y .

$$(\forall x) [F(x) \rightarrow (\exists y) [C(y) \wedge O(x,y)]] \wedge$$

$$(\forall x)(\forall y) [D(x) \wedge C(y) \rightarrow O(x,y)]' \rightarrow$$

$$(\forall x) (D(x) \rightarrow F(x))'$$

1. $(\forall x)[F(x) \rightarrow (\exists y)[C(y) \wedge O(x, y)]]$ h, γ
2. $(\forall x)(\forall y)[D(x) \wedge C(y) \rightarrow O(x, y)]$ h, γ
3. $F(x) \rightarrow (\exists y)[C(y) \wedge O(x, y)]$ $1, u_i$
4. $F(x)$ $temp\ h, \gamma$
5. $(\exists y)[C(y) \wedge O(x, y)]$ $3, 4, m_f$
6. $C(y) \wedge O(x, y)$ $5, e_i$
7. $O(x, y)$ $6, sing$
8. $D(x) \wedge C(y) \rightarrow O(x, y)'$ $u_i (twice)$
9. $(D(x) \wedge C(y))'$ $7, 8\ mt$
10. $D(x)' \vee C(y)$ $9, de\ Morgan$
11. $C(y)$ $6, sing$
12. $(C(y)*)'$ $11, dn$
13. $D(x)'$ $10, 12, ds$
14. $F(x) \rightarrow D(x)'$ $temp\ h, \gamma\ discharged$
15. $(D(x))' \rightarrow (F(x))$ $14, contraposition$
16. $D(x) \rightarrow F(x)$ $15, dn$
17. $(\forall x)[D(x) \rightarrow F(x)]$ $16, u_{\forall}$