MAT115 Exam 1 (Spring 2022)

Name:

Directions: Show your work! Cross out – don't erase. Some useful info:

a. First few primes: 2 3 5 7 11 13 17 19 23 29 31 37

b. First few powers of 2: 1 2 4 8 16 32 64 128 256 512

c. Formula for two-toss sampling: $r = 2\left(r_v - \frac{1}{4}\right)$

Problem 1: (24 pts) Your number is 228. For each of the three problems below, draw the appropriate tree and then put your final result at the bottom, as appropriate for each situation.

Prime Factorization	Binary Factorization	Primitive Counting

228 =	228 =	tally stick:

Problem 2: (20 pts) Answer four of the following six (writing "skip" on the other two).

a. We can think of the tetrahedron **as a graph** – draw it, and describe it using graph terminology.

b. Describe one-to-one correspondence, using the Furry Arms Hotel episode as an example.

c. How did we know that there is an error in Yanghui's triangle?

d. How did Vi Hart use one hand to count from 1 to 31, while hand-dancing?

e. Draw a pair of complete graphs with three vertices, one "balanced", and one "unbalanced" (per Strogatz's chapter *The Enemy of My Enemy*). **Indicate which is which!**

f. Speaking of pairs, what's special about the pair 179 and 181?

Problem 3: (14 pts) The Great Fraudini

a. (5 pts) Explain how the Great Fraudini's trick works. How can Fraudini "read minds?" In particular, what mathematical fact makes it work? (It begins "Every counting number....")

b. (9 pts) With six cards (as we used in class), each card has 32 numbers on it. If Fraudini added two new cards (and updated the old ones) so that he could "read" your mind for larger numbers,

   i. what numbers would appear in the upper left-hand corners of the new cards? Explain.

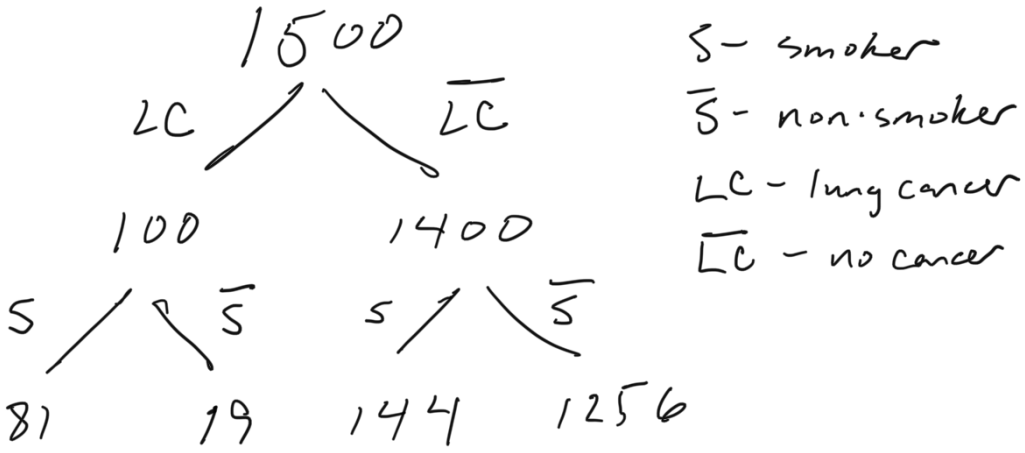
   ii. what would be the largest number that he could "read" (it was 63 for six cards)? Explain.

   iii. which of his eight cards would have the number 213 on them? Explain.

Problem 4: (15 pts) Some cancer statistics:

- Men who smoke are 24 times more likely to develop lung cancer than men who don't.
- The lifetime risk of developing lung cancer for men in the U.S. is 1 in 15.

Based on this information, I naturally frequency-wise chose to start with 1500 men, and built this tree, which should help us to compute some conditional probabilities:



a. Given that a male is a smoker, what's the chance that they have lung cancer? (Show work!)

b. Given that a male is a **non**-smoker, what's the chance that they have lung cancer?

c. Given that a male is positive for lung cancer, what's the chance that they're a smoker?

Problem 5: (12 pts)

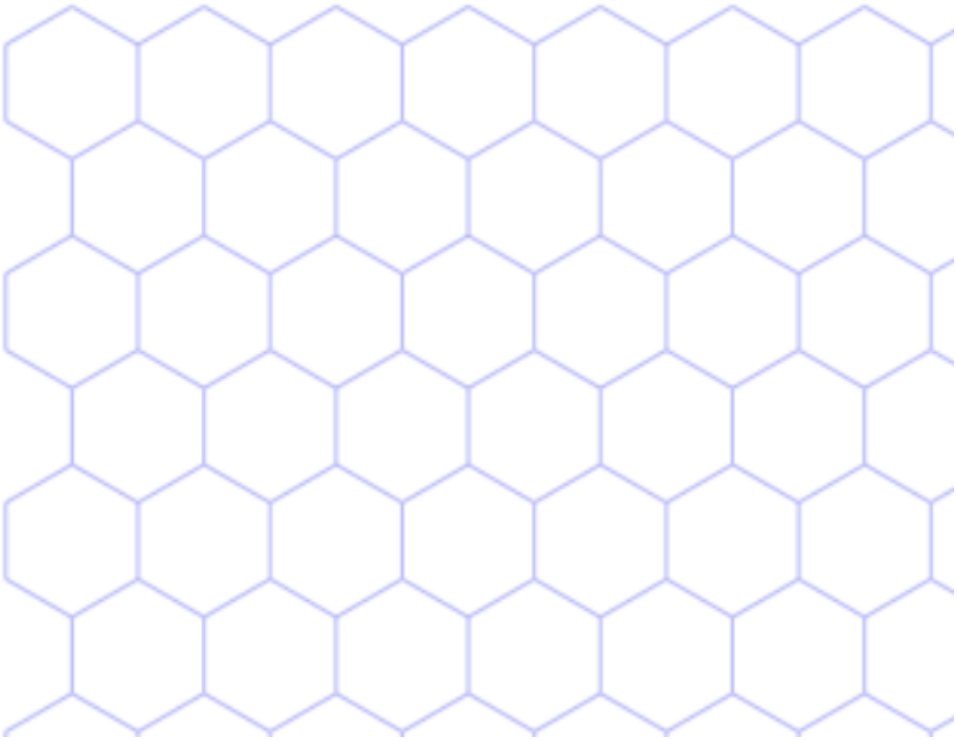
a. (6 pts) Draw a labelled, directed graph to illustrate an SIR model for covid infection. Include as many details (e.g. what do S, I, and R stand for?) as you can, to create a better model.

b. (6 pts) Suppose that we want to estimate the true rate r of covid-positive people in a population (where they are reluctant to disclose their status). We use the two-coin-toss method, and those who toss two heads will lie about their status, reporting the **opposite** status. After flipping their coins, 32% of the people report that they are covid-positive.

What is our estimate for the rate r of covid-positive people in this population?

Problem 6: (15 pts)

a. (5 pts) Create six rows of Pascal's triangle. (This hexagonal grid may help organize.)



b. (4 pts) Demonstrate on your triangle how the

- triangular numbers and
- powers of 2

appear in Pascal's triangle in a systematic way.

c. (3 pts) How many distinctly different Facebooks can you create that have exactly three distinct individuals in them, named A, B, and C? Explain.

d. (3 pts) You can invite three of your five friends to a party. How many ways could you make the choice? Explain.