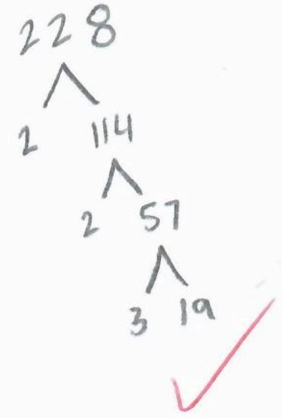
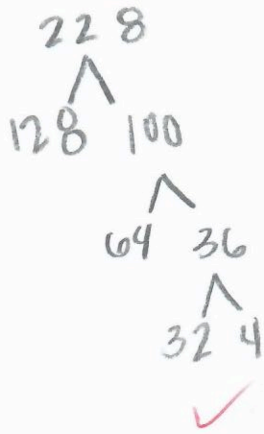
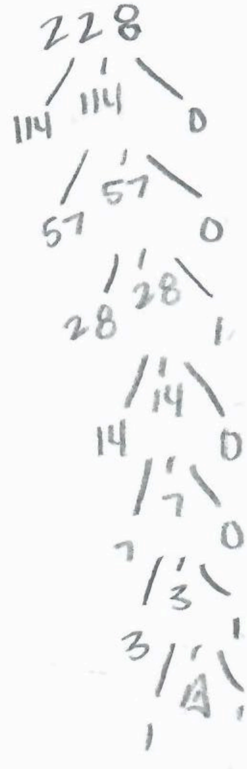
Directions: Show your work! Cross out – don't erase. Some useful info:

a. First few primes: 2 3 5 7 11 13 17 19 23 29 31 37

b. First few powers of 2: 1 2 4 8 16 32 64 128 256 512

c. Formula for two-toss sampling: $r = 2\left(r_v - \frac{1}{4}\right)$

Problem 1: (24 pts) Your number is 228. For each of the three problems below, draw the appropriate tree and then put your final result at the bottom, as appropriate for each situation.

Prime Factorization	Binary Factorization	Primitive Counting
		

228 = 2, 2, 3, 19	228 = 4, 32, 64, 128 $2^2+2^5+2^6+2^7$	tally stick: 11100100

228 ≟ 2*2*3*19

Problem 2: (20 pts) Answer four of the following six (writing "skip" on the other two).

a. We can think of the tetrahedron **as a graph** – draw it, and describe it using graph terminology.

Not sure about that one. Can't we unbalance it?

A tetrahedron is balenced, connected, and complete ✓ graph.

b. Describe one-to-one correspondence, using the Furry Arms Hotel episode as an example.

One-to-one correspondence is when items relate to a set value. The Furry Arms Hotel episode shows one-to-one correspondence in the word fish for every fish said from a pengoin. ✓

Sort of... Each Penguin is matched to their choice.

c. How did we know that there is an error in Yanghui's triangle?

Yangnui's triangle was symetrical up until the error, making the sum of the row not equal to a power of 2.

Good

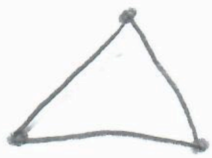
d. How did Vi Hart use one hand to count from 1 to 31, while hand-dancing?

Vi Hart labeled each finger a power of 2, combining fingers by placing them on the table.

Yep!

e. Draw a pair of complete graphs with three vertices, one "balanced", and one "unbalanced" (per Strogatz's chapter *The Enemy of My Enemy*). **Indicate which is which!**

unbalenced



balenced

good

f. Speaking of pairs, what's special about the pair 179 and 181?

179 and 181 are twin primes. ✓
lone pairs

Problem 3: (14 pts) The Great Fraudini

Great!

a. (5 pts) Explain how the Great Fraudini's trick works. How can Fraudini "read minds?" In particular, what mathematical fact makes it work? (It begins "Every counting number....")

Fraudini has 6 cards, all with powers of 2 in the upper left-hand corners. He asks you to pick a number 1-63 + shows you the 6 cards, asking you which cards your number appears on. He can always "read minds" because your number will be the sum of the powers of 2 in the upper left-hand corners. Every counting number is a power of 2 or the sum of powers of 2 which makes this trick work every time. distinct

b. (9 pts) With six cards (as we used in class), each card has 32 numbers on it. If Fraudini added two new cards (and updated the old ones) so that he could "read" your mind for larger numbers,

i. what numbers would appear in the upper left-hand corners of the new cards? Explain.

✓ The numbers 64 + 128 would appear in the upper left-handed corners because Fraudini is able to "read minds" by adding up the powers of 2 that your number is on the card of + 64 + 128 are the next powers of 2 $(2^6, 2^7)$.

ii. what would be the largest number that he could "read" (it was 63 for six cards)? Explain.

✓ The largest number that he could "read" would be 255 because that is the sum of the 1st 8 powers of 2. Similarly, the 1st 6 powers of 2 add up to 63, which was the largest number Fraudini could "read" for 6 cards.

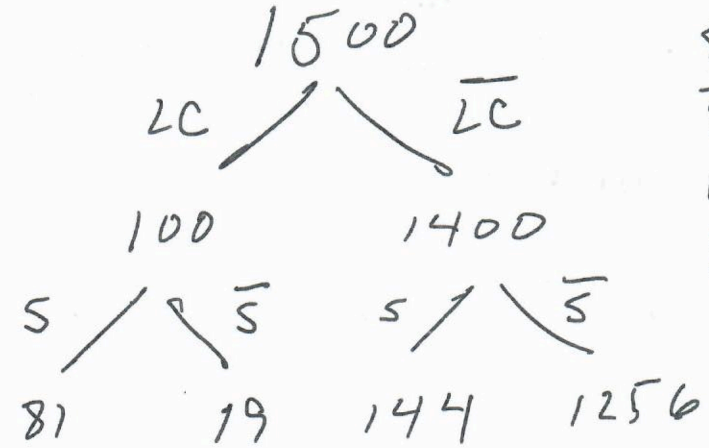
iii. which of his eight cards would have the number 213 on them? Explain.

✓ 213 would appear on cards 1, 4, 16, 64 + 128 because $2^0 + 2^2 + 2^4 + 2^6 + 2^7 = 213$ + the powers of 2 are the reason his trick works.

Problem 4: (15 pts) Some cancer statistics:

- Men who smoke are 24 times more likely to develop lung cancer than men who don't.
- The lifetime risk of developing lung cancer for men in the U.S. is 1 in 15.

Based on this information, I naturally frequency-wise chose to start with 1500 men, and built this tree, which should help us to compute some conditional probabilities:



S – smoker
$\overline{S}$ – non-smoker
LC – lung cancer
$\overline{LC}$ – no cancer

a. Given that a male is a smoker, what's the chance that they have lung cancer? (Show work!)

$\frac{81}{81+144} = \frac{81}{225} = 0.36$

He would have a 0.36 or 36% chance of having lung cancer

b. Given that a male is a **non**-smoker, what's the chance that they have lung cancer?

$\frac{19}{19+1256} = \frac{19}{1275} = 0.014$

He would have a 0.014 or 1.4% chance of having lung cancer.

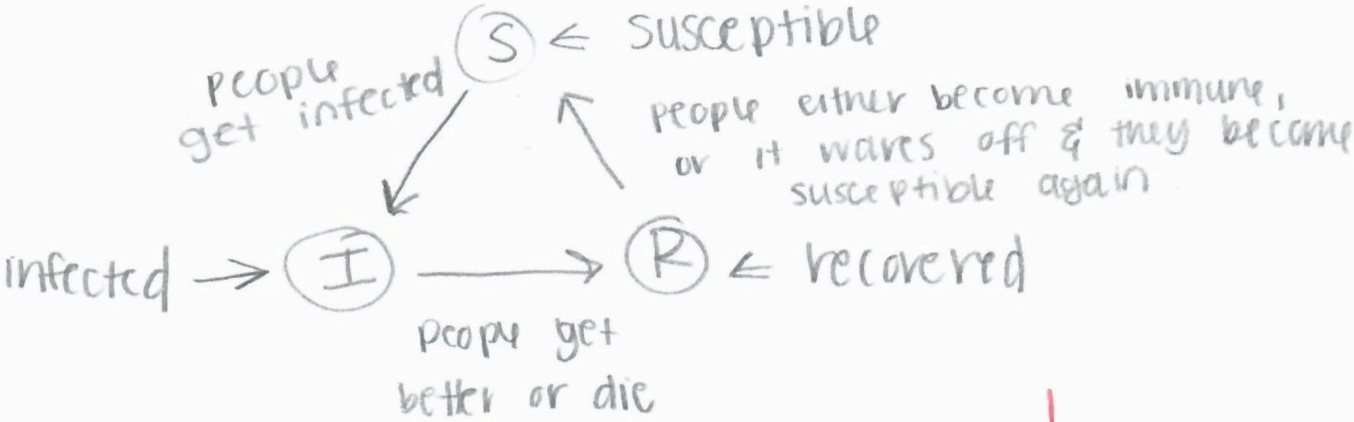
c. Given that a male is positive for lung cancer, what's the chance that they're a smoker?

$\frac{81}{100} = .81$

He would have an .81 or 81% chance of being a smoker.

Problem 5: (12 pts)

a. (6 pts) Draw a labelled, directed graph to illustrate an SIR model for covid infection. Include as many details (e.g. what do S, I, and R stand for?) as you can, to create a better model.



Good

b. (6 pts) Suppose that we want to estimate the true rate r of covid-positive people in a population (where they are reluctant to disclose their status). We use the two-coin-toss method, and those who toss two heads will lie about their status, reporting the **opposite** status. After flipping their coins, 32% of the people report that they are covid-positive.

What is our estimate for the rate r of covid-positive people in this population?

$r = 2(rv - \frac{1}{4})$

$r = 2(\frac{32}{100} - \frac{1}{4})$

$r = 2(\frac{32}{100} - \frac{25}{100})$

$r = 2(\frac{7}{100}) = 0.14$

$r \approx 0.14$

or $\approx 14\%$.

Problem 6: (15 pts)

a. (5 pts) Create six rows of Pascal's triangle. (This hexagonal grid may help organize.)

1 – 1

1 1 – 2

1 2 1 – 4

1 3 3 1 – 8

1 4 6 4 1 – 16

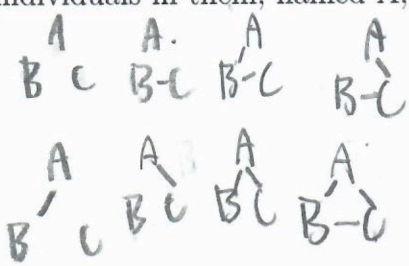
1 5 10 10 5 1 – 32

b. (4 pts) Demonstrate on your triangle how the

- triangular numbers and
- powers of 2

appear in Pascal's triangle in a systematic way.

c. (3 pts) How many distinctly different Facebooks can you create that have exactly three distinct individuals in them, named A, B, and C? Explain.



8

Good

d. (3 pts) You can invite three of your five friends to a party. How many ways could you make the choice? Explain.

10 different ways to make the choice. I know this from looking at the 5th row of pascal's triangle and counting over 3.

Good