

# MAT115 Exam 2 (Spring 2022)

Name:

**Directions: Show your work.** Answers without justification may result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer. Good luck!

**Problem 1:** (24 pts) Write each of the following numbers in Babylonian and in Mayan (show your work):

| Number | Babylonian | Mayan |
|--------|------------|-------|
| 59     |            |       |
| 367    |            |       |
| 7200   |            |       |
| 10840  |            |       |

**Problem 2:** (10 pts) Translate the following Babylonian tablet, filling in the missing numbers:

| row | left number | right number |
|-----|-------------|--------------|
| 1   |             |              |
| 2   |             |              |
| 3   |             |              |
| 4   |             |              |
| 5   |             |              |
| 6   |             |              |
| 7   |             |              |
| 8   |             |              |
| 9   |             |              |
| 10  |             |              |
| 11  |             |              |





**Problem 4:** (16 pts)

a. (12 pts) Fibonacci Nim. You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to

- explain why you would rather go first or second, and
- then give your **first** move (assuming that you were player one), and explain exactly why you made it. If you're in a bad situation, play the slow-down strategy.

| Number | Player 1 or 2? | As player 1, what would be your first move? |
|--------|----------------|---|
| 107    |                |   |
| 55     |                |   |
| 42     |                |   |

b. (4 pts) Ratios of successive Fibonacci numbers approach what number as they get larger and larger? What is the number's name and value?

**Problem 5:** (20 pts) Demonstrate Egyptian multiplication and division:

$$29 * 54 =$$

$$19 * 73 =$$

$$\frac{23}{32} =$$

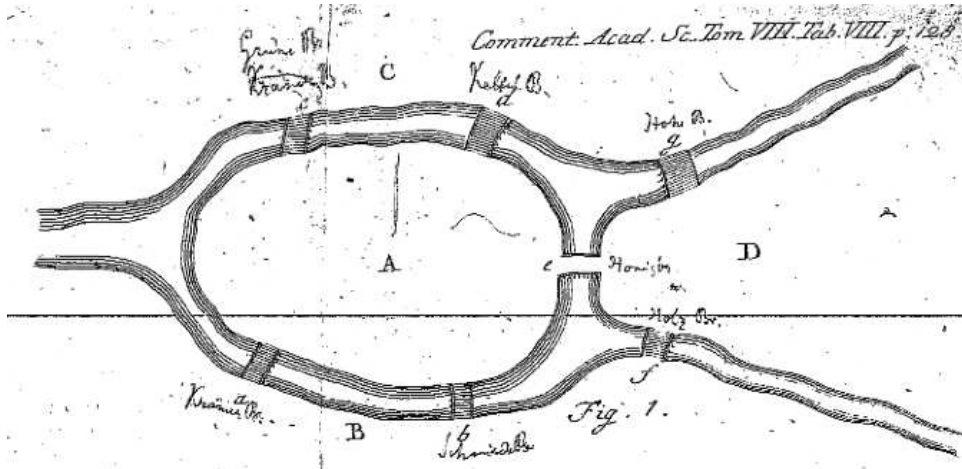
$$\frac{28}{25} =$$

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A short  $2/n$  table from the Rhind Mathematical Papyrus

|                      |                                      |                               |
|----------------------|--------------------------------------|-------------------------------|
| $2/3 = 1/2 + 1/6$    | $2/5 = 1/3 + 1/15$                   | $2/7 = 1/4 + 1/28$            |
| $2/9 = 1/6 + 1/18$   | $2/11 = 1/6 + 1/66$                  | $2/13 = 1/8 + 1/52 + 1/104$   |
| $2/15 = 1/10 + 1/30$ | $2/17 = 1/12 + 1/51 + 1/68$          | $2/19 = 1/12 + 1/76 + 1/114$  |
| $2/21 = 1/14 + 1/42$ | $2/23 = 1/12 + 1/276$                | $2/25 = 1/15 + 1/75$          |
| $2/27 = 1/18 + 1/54$ | $2/29 = 1/24 + 1/58 + 1/174 + 1/232$ | $2/31 = 1/20 + 1/124 + 1/155$ |

**Problem 6:** (10 pts) Below is the situation Euler considered when he single-handedly invented graph theory. How did Euler ruin the fun of the citizens of Königsberg?



In particular,

- a. (4 pts) Turn this image into a graph, explaining Euler's essential ideas.
  
- b. (4 pts) Explain what Euler determined as essential for the existence of an "Euler path", and how he knew that this graph didn't have one.
  
- c. (2 pts) This graph is planar. Verify that the Euler formula holds. How many regions  $r$ , arcs  $a$ , and nodes  $n$  are there?