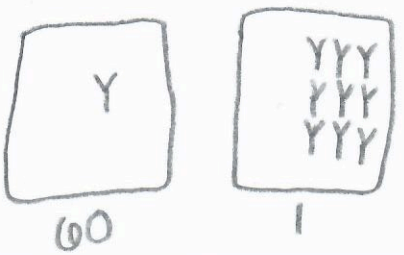
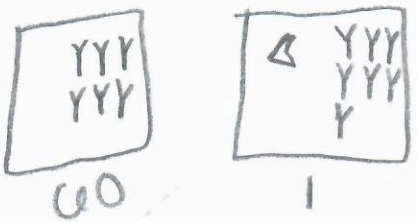
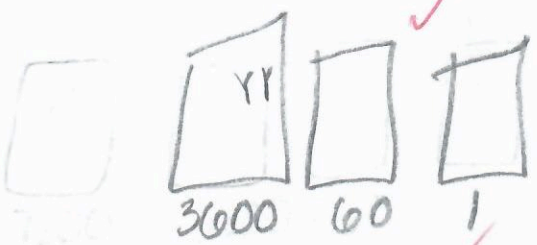
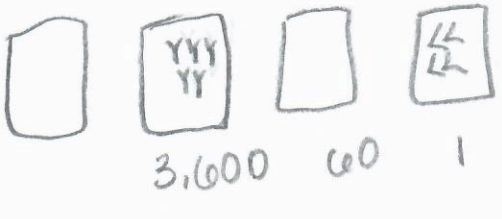
Directions: **Show your work**. Answers without justification may result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer. Good luck!

Problem 1: (24 pts) Write each of the following numbers in Babylonian and in Mayan (show your work):

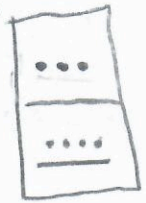
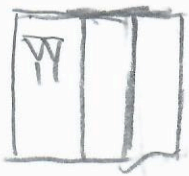
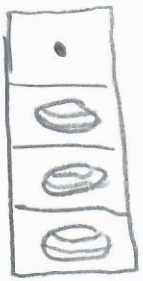
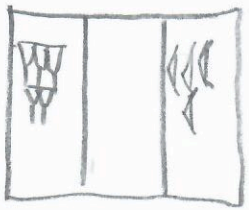
Number	Babylonian	Mayan
69		
377		
7200		
18040		

24

Problem 1: (24 pts) Write each of the following numbers in Babylonian and in Mayan (show your work):

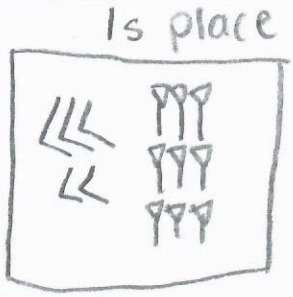
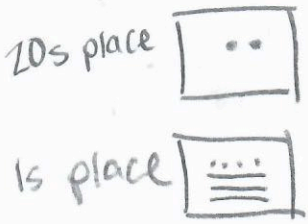
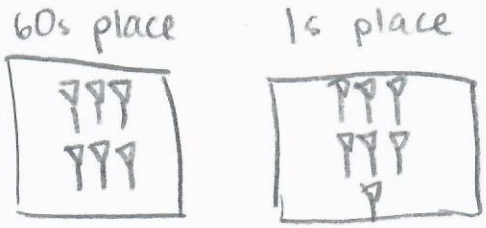
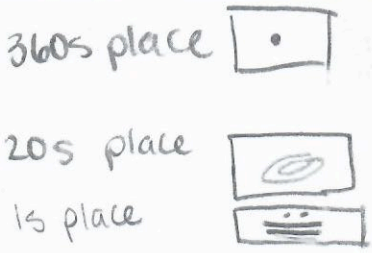
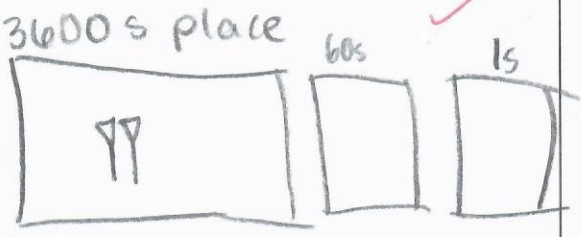
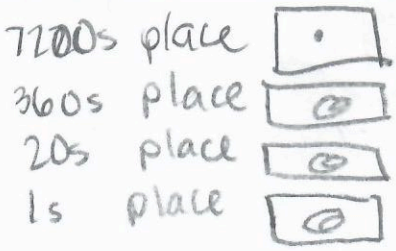
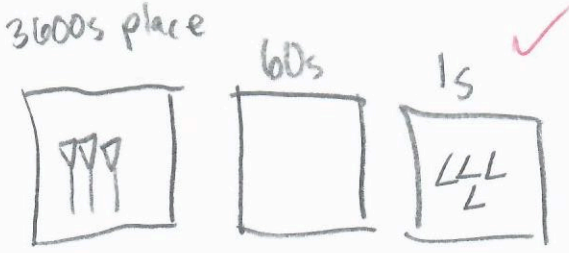
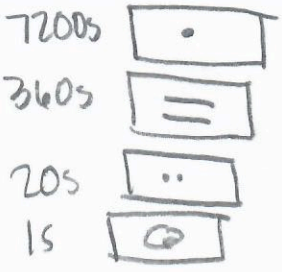
1, 60, 3600

1, 20, 360

Number	Babylonian	Mayan
69	 ✓	 ✓
377	 ✓	 ✓
7200	 ✓	 ✓
18040	 ✓	 ✓

R = e − v + 2

1.618

Number	3600 60 1 Babylonian	7200 360 20 1 Mayan
59	 ✓	 ✓
367	 ✓	 ✓
7200	 ✓	 ✓
10840	 ✓	 ✓

Problem 2: (10 pts) Translate the following Babylonian tablet, filling in the missing numbers:

row	left number	right number
1	6	36
2	7	49
3	8	64
4	9	81
5	10	110
6	11	121
7	12	144
8	13	169
9	14	196
10	15	225
11	16	256

Problem 3: (20 pts) Short answer:

a. What is the characteristic that makes the golden rectangle golden?

If a square is taken out of it, it remains proportional to the original rectangle.

b. To what does the title "Working your Quads" refer?

Work the quadradic formula

c. Share one detail from "The History of Mathematics", By Anne Rooney, concerning a mathematical contribution from a civilization other than the Mayans, Babylonians, or Egyptians.

The numerical system we use today comes from ~~modern-day~~ ancient! India & was introduced to Europe through Islamic scholars.

d. What is the thing that neuroscientists postulate humans may be able to do that baboons cannot (from "Is Geometry a Language That Only Humans Know?")?

Humans can recognize & gravitate towards right angles & geometrically consistent shapes.

Problem 4: (16 pts)

a. (12 pts) Fibonacci Nim. You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to

- explain why you would rather go first or second, and
- then give your **first** move (assuming that you were player one), and explain exactly why you made it. If you're in a bad situation, play the slow-down strategy.

Number	Player 1 or 2?	As player 1, what would be your first move?
107	1st because 107 is not Fibonacci ✓	take 5 because it is the smallest Fibonacci number ✓ that creates 107, 89+13+(5)
55	2nd because 55 is Fibonacci ✓	take 1 to slow the game down + get back in the drivers seat ✓
42	1st because 42 is not Fibonacci ✓	take 8 because it is the smallest Fibonacci number ✓ that creates 42, 34+(8)

107 → 89, 18 → 13, 5

42 → 34, 8

b. (4 pts) Ratios of successive Fibonacci numbers approach what number as they get larger and larger? What is the number's name and value?

they approach infinity $\frac{1+\sqrt{5}}{2}$ = golden ratio ✓

16

Well done

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 136 144

Problem 4: (16 pts)

a. (12 pts) Fibonacci Nim. You and I are playing a game of Fibonacci Nim with a given number of pieces of candy. In each of the three cases below, we start with the number of candies specified. You are to

- explain why you would rather go first or second, and
- then give your **first** move (assuming that you were player one), and explain exactly why you made it. If you're in a bad situation, play the slow-down strategy.

Number	Player 1 or 2?	As player 1, what would be your first move?
107	1 because it is not Fibonacci	107 → 89, 18 → 13 ☒ ✓ 5 because it is the smallest Fibonacci
55	2nd because it is Fibonacci ✓	1 because you want to hope the other player messes up ✓
42	1 because it is not Fibonacci	42 → 34 ☒ ✓ 8 because it is the smallest Fibonacci

Problem 5: (20 pts) Demonstrate Egyptian multiplication and division:

19*73=

1	73	x
2	146	x
4	292	
8	584	
16	1168	x

19 = 16 + 3 = 16 + 2 + 1

73 + 146 + 1168 = 1387

29*54=

1	54	x
2	108	
4	216	x
8	432	x
16	864	x

29 = 16 + 13 = 16 + 8 + 5 = 16 + 8 + 4 + 1

54 + 216 + 432 + 864 = 1566

$\frac{28}{25} = \frac{25}{25} + \frac{3}{25} = 1 + \frac{3}{25} = 1 + \frac{2}{25} + \frac{1}{25}$

$= 1 + \frac{1}{15} + \frac{1}{75} + \frac{1}{25}$

$= 1 + \frac{1}{15} + \frac{1}{25} + \frac{1}{75}$

$\frac{23}{32} = \frac{16+4+2+1}{32} = \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

Problem 5: (20 pts) Demonstrate Egyptian multiplication and division:

29*54=

1	54	*
2	108	
4	216	*
8	432	*
16	864	*

54+216+432+864=1566

19*73=

1	73	*
2	146	*
4	292	
8	584	
16	1168	*

73+146+1168=1387

$\frac{23}{32}$ = $\frac{1}{2}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$

23-7=16

$\frac{16}{32}=\frac{1}{2}$

$\frac{7}{32}=\frac{3}{32}+\frac{4}{32}$

$\frac{4}{32}=\frac{1}{8}$

$\frac{3}{32}=\frac{1}{32}+\frac{2}{32}$

$\frac{2}{32}=\frac{1}{16}$

$\frac{28}{25}$ = $1+\frac{1}{15}+\frac{1}{25}+\frac{1}{75}$

28-3=25

$\frac{25}{25}=1$

$\frac{3}{25}=\frac{1}{25}+\frac{2}{25}$

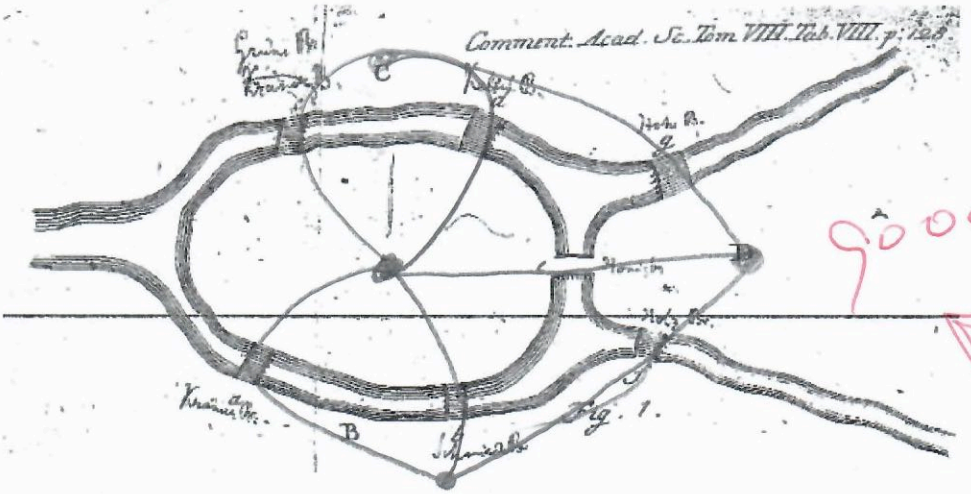
$\frac{2}{25}=\frac{1}{15}+\frac{1}{75}$

A short 2/n table from the Rhind Mathematical Papyrus

```
 2/3 = 1/2 + 1/6        2/5 = 1/3 + 1/15                2/7 = 1/4 + 1/28
 2/9 = 1/6 + 1/18      2/11 = 1/6 + 1/66               2/13 = 1/8 + 1/52 + 1/104
2/15 = 1/10 + 1/30     2/17 = 1/12 + 1/51 + 1/68       2/19 = 1/12 + 1/76 + 1/114
2/21 = 1/14 + 1/42     2/23 = 1/12 + 1/276                 2/25 = 1/15 + 1/75
2/27 = 1/18 + 1/54     2/29 = 1/24 + 1/58 + 1/174 + 1/232  2/31 = 1/20 + 1/124 + 1/155
```

20 Well done

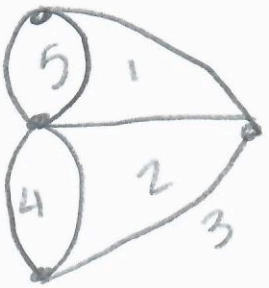
Problem 6: (10 pts) Below is the situation Euler considered when he single-handedly invented graph theory. How did Euler ruin the fun of the citizens of Konigsberg?



good

In particular,

a. (4 pts) Turn this image into a graph, explaining Euler's essential ideas.



handshake says pairs are never an odd number of odds

b. (4 pts) Explain what Euler determined as essential for the existence of an "Euler path", and how he knew that this graph didn't have one.

Euler determined the ~~handshake~~ theroem which says that a graph may not have more than two odd vertices in order to have an Euler path. This graph has 4 odd vertices so he knew it did not.

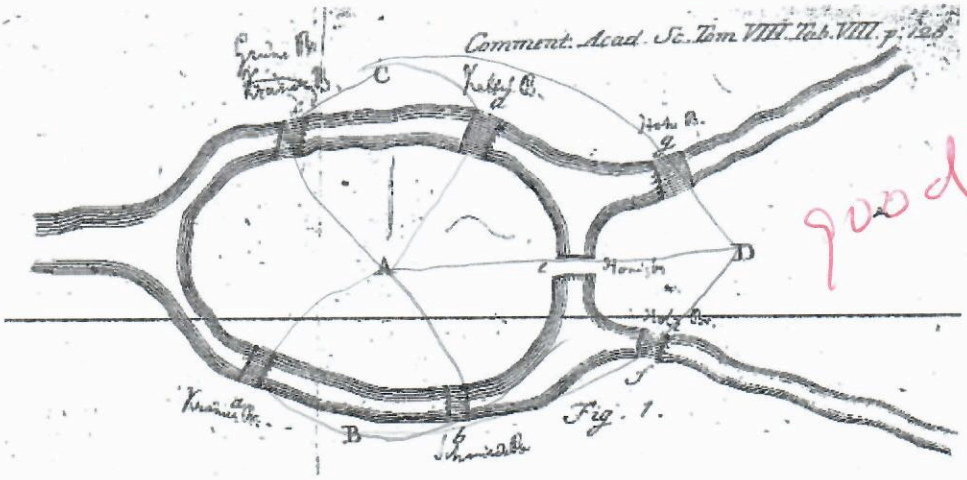
c. (2 pts) This graph is planar. Verify that the Euler formula holds. How many regions r, arcs a, and nodes n are there?

$R - A + N = 2$

$5 - 7 + 4 = 2$

$R = 5$
$A = 7$
$N = 4$

Problem 6: (10 pts) Below is the situation Euler considered when he single-handedly invented graph theory. How did Euler ruin the fun of the citizens of Konigsberg?



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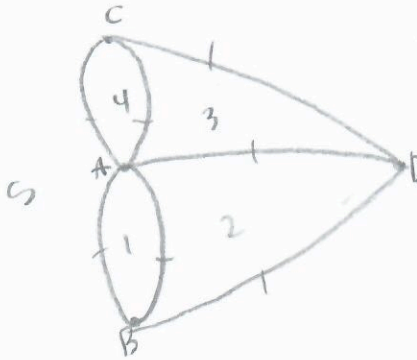

There is no way to go through each bridge once & hit every single stop.

b. (4 pts) Explain what Euler determined as essential for the existence of an "Euler path", and how he knew that this graph didn't have one.

It is essential for zero or two vertices to have an odd degree to be an Euler path. This graph has four odd degrees, therefore it isn't an Euler path.

c. (2 pts) This graph is planar. Verify that the Euler formula holds. How many regions r, arcs a, and nodes n are there? A planar graph has an $r-a+n=2$, this graph has 5 regions, 7 arcs, & 4 nodes. $5-7+4=2$ therefore, this is a planar graph.