Derivative as Function Worksheet

Limit Laws

If $\lim\limits_{x\to a} f(x) = M$ and $\lim\limits_{x\to a} g(x) = N$

- **Sum** $\lim\limits_{x\to a}[f(x)+g(x)] = M+N$
- **Difference** $\lim\limits_{x\to a}[f(x)-g(x)] = M-N$
- **Constant** $\lim\limits_{x\to a}[k\cdot f(x)] = k\cdot M$
- **Product** $\lim\limits_{x\to a}[f(x)g(x)] = M\cdot N$
- **Quotient** $\lim\limits_{x\to a}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{M}{N}$ $\quad N \neq 0$
- **Power** $\lim\limits_{x\to a}[f(x)]^n = M^n$ $\quad$ *n* is a positive integer
- **Root** $\lim\limits_{x\to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{M}$ $\quad$ *n* is a positive integer

1. Using the limit definition of the derivative, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, as well as the limit laws, to find the derivative of $f(x) = 3x^2 + x - 5$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left(-5+h+x+3\,(h+x)^2\right)-\left(-5+x+3\,x^2\right)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\left(-5+h+3\,h^2+x+6\,h\,x+3\,x^2\right)-\left(-5+x+3\,x^2\right)}{h} = \lim_{h\to 0}\frac{h+3\,h^2+6\,h\,x}{h} f'(x)$$

$$f'(x) = \underbrace{\lim_{h\to 0}\,(1+3\,h+6\,x)}_{\text{we can safely pass to the limit: let } h\to 0,} = 6x+1$$

every term has an h in it - cancel one term from each

2. Using the limit definition of the derivative, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, as well as the limit laws, to find the derivative of $f(x) = \frac{1}{x+5}$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left(\frac{1}{5+h+x}\right)-\left(\frac{1}{5+x}\right)}{h}$$

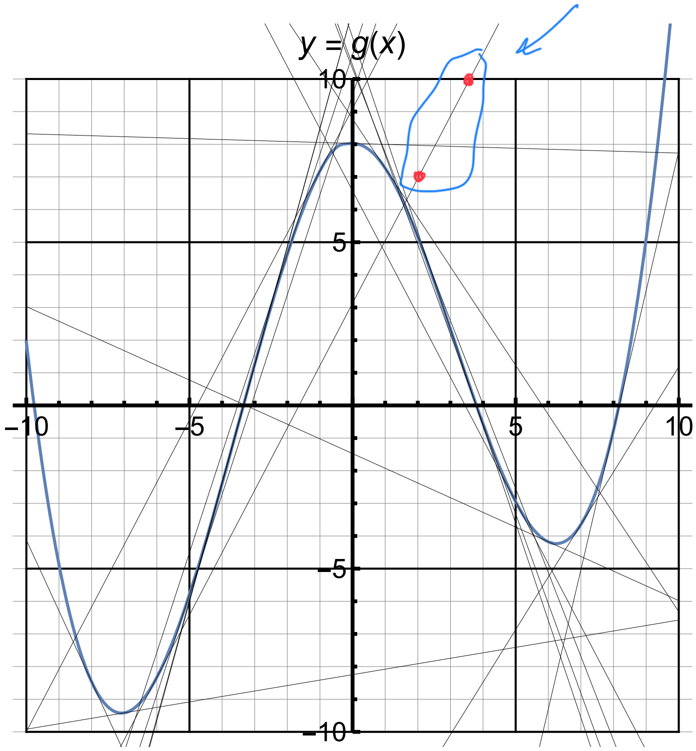
$$f'(x) = \lim_{h\to 0}\left(-\frac{\cancel{h}}{(5+x)\,(5+h+x)}\right)/\cancel{h} = \underbrace{\lim_{h\to 0}\frac{-1}{(5+x)\,(5+h+x)} = -\frac{1}{(5+x)^2}}_{\text{we can safely pass to the limit: let } h\to 0.}$$

$m \approx \frac{3}{1.5} = 2$ at $x = -6$

Check table below for actual value.

3. Below is the graph of function $g(x)$.

Out[136]=



Look at all those tangent lines! Of course you really don't have to draw them - you just need their slopes. Hold the ruler up, & measure rise over run based on two points.

3.1. By using some kind of a straight edge, for each of the following x-values draw an estimate of the tangent line and approximate its slope using rise over run. Each of these slopes is the value of the derivative at that x-value.

Out[40]//TableForm=

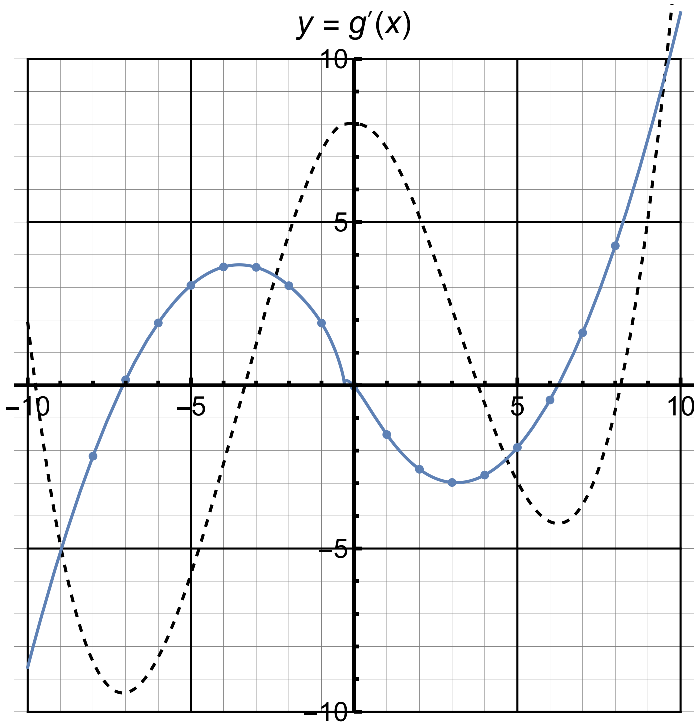
	-8	-7	-6	-5	-4	-3	-2	-1	0
x	-8	-7	-6	-5	-4	-3	-2	-1	0
g'(x)	-2.17	0.17	1.91	3.06	3.63	3.61	3.05	1.91	-0.03

Out[41]//TableForm=

	1	2	3	4	5	6	7	8
x	1	2	3	4	5	6	7	8
g'(x)	-1.51	-2.57	-2.98	-2.75	-1.9	-0.45	1.61	4.27

3.2. Plot these $(x, g'(x))$ and connect them to get the graph $y = g'(x)$.

Out[104]=



4. (textbook, #9) For each graph that provides an original function y=f(x) in the following figures, your task is to sketch an approximate graph of its derivative function, $y=f'(x)$, on the axes immediately below. View the scale of the grid for the graph of f as being 1×1, and assume the horizontal scale of the grid for the graph of f' is identical to that for f. If you need to adjust the vertical scale on the axes for the graph of f', you should label that accordingly.