# Nonconstant Rate Worksheet

## Corresponding to Section 1.1

**1.** Use the following variable representations:
$x$ = hours gone by
$y = f(x)$ = miles traveled as of time $x$ (the odometer reading at time $x$)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $y = f(x)$ | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 200 | 300 |
| Average speed in last hour | -- | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |

**1.1.** What is the average speed from $x = 0$ to $x = 8$?

```
In[338]:= (300 - 100) / 8
Out[338]= 25
```

**1.2.** Fill in the bottom row with the average speed over the last hour.

See table

**1.3.** If the speed limit on the road being traveled is 65 mph, is the average speed within the speed limit?

Absolutely

**1.4.** If a police car stops the car after 6.5 hours, will the driver receive a ticket (based on the average speed)?

Yes.

**1.5.** Does the average velocity over 8 hours matter when giving speeding tickets? When would it be enough to convict, and when would it not?

If the average speed were above the actual speed limit, then we could be sure that the vehicle was driven above the speed limit at some point -- so we could issue a ticket. Otherwise, no.

**2.** Use the following variable representations:
$x$ = hours gone by
$y = f(x)$ = total money earned after $x$ hours

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $y = f(x)$ | 10 | 20 | 30 | 40 | 50 | 50 | 50 | 50 | 50 |
| salary at time $x$ in dollars per hour | 10 | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 |

**2.1.** What is the average salary from $x = 0$ to $x = 8$?

```
In[340]:= (50 - 0) / 8.0
Out[340]= 6.25
```

**2.2.** If the legal minimum wage is $7.50 per hour, does the average salary over the 8 hour period meet this requirement?

NO.

**2.3.** Does the average salary over 8 hours matter when assessing the minimum wage requirement?

Only if someone is actually working during those 8 hours.

**2.4.** Fill in the last row of the table with your best guess of the wage being earned per hour at time $x$.

**2.5.** Explain how you are making your best guess of the wage being earned.

We're assuming that they're actually working during the hour reported.

**3.** You are given that the height in feet of a ball at time $t$ seconds is $h(t) = 100 - 10\,t - 16\,t^2$.

In[341]:=
```
h[t_] := 100 - 10 t - 16 t^2
```

**3.1.** Find the average velocity over the following time intervals.

- [0, 0.3]

In[343]:=
```
t0 = 0.3;
(h[t0] - h[0]) / (t0 - 0)
```

Out[344]= -14.8

- [0, 0.2]

In[345]:=
```
t0 = 0.2;
(h[t0] - h[0]) / (t0 - 0)
```

Out[346]= -13.2

- [0, 0.1]

In[347]:=
```
t0 = 0.1;
(h[t0] - h[0]) / (t0 - 0)
```

Out[348]= -11.6

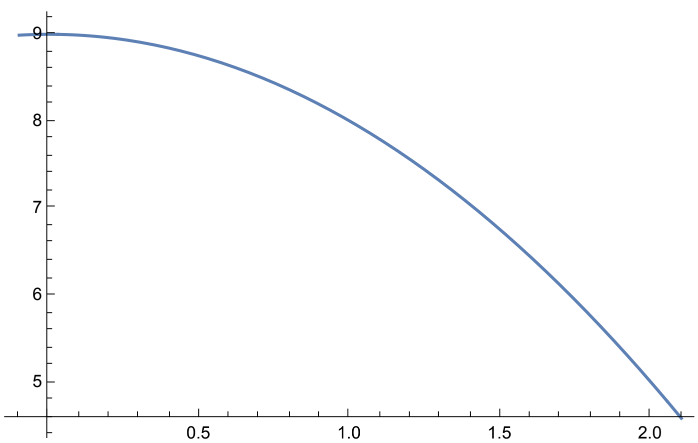
**3.2.** What is the best approximation available for the instantaneous velocity of the ball when $t = 0$?

In[349]:=
```
t0 = 0.1;
(h[t0] - h[0]) / (t0 - 0)
```

Out[350]= -11.6

**4.** Given that $f(x) = 9 - x^2$, use a graphing device (like Desmos) to graph $f$.

In[357]:=
```
f[x_] := 9 - x^2
AV[a_] := (f[a] - f[0]) / a
p1 = Plot[f[x], {x, -.1, 2.1}]
```

Out[359]=


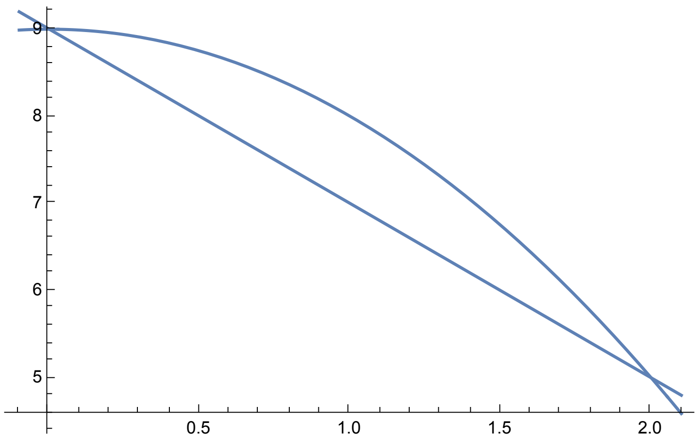

**4.1.** Along with this graph plot the secant line for each of the given intervals and find the secant line's slope.

- [0, 2]

In[367]:=
```
x0 = 2;
AV[x0]
se1[x_] := f[0] + AV[x0] (x - 0)
Show[p1, Plot[se1[x], {x, -.1, 2.1}]]
```

Out[368]= -2

Out[370]=

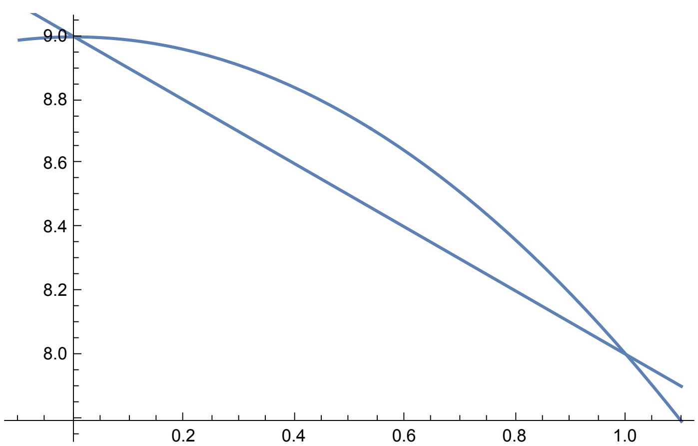


- [0, 1]

In[376]:=
```
x0 = 1;
AV[x0]
se1[x_] := f[0] + AV[x0] (x - 0)
p1 = Plot[f[x], {x, -.1, x0 + .1}];
Show[p1, Plot[se1[x], {x, -.1, x0 + .1}]]
```

Out[377]= -1

Out[380]=

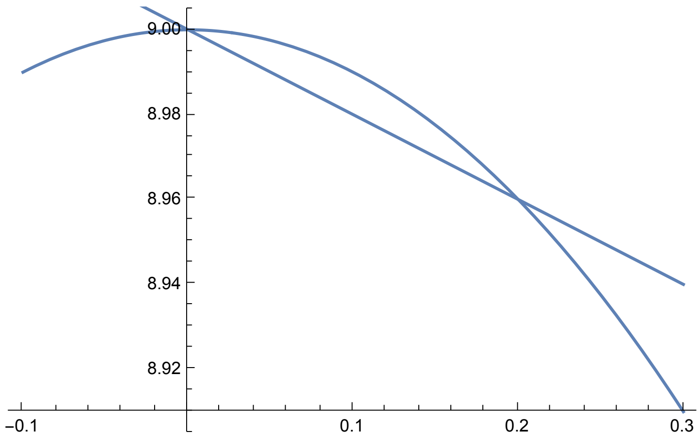


- [0, 0.2]

In[381]:=
```
x0 = 0.2;
AV[x0]
se1[x_] := f[0] + AV[x0] (x - 0)
p1 = Plot[f[x], {x, -.1, x0 + .1}];
Show[p1, Plot[se1[x], {x, -.1, x0 + .1}]]
```

Out[382]= -0.2

Out[385]=

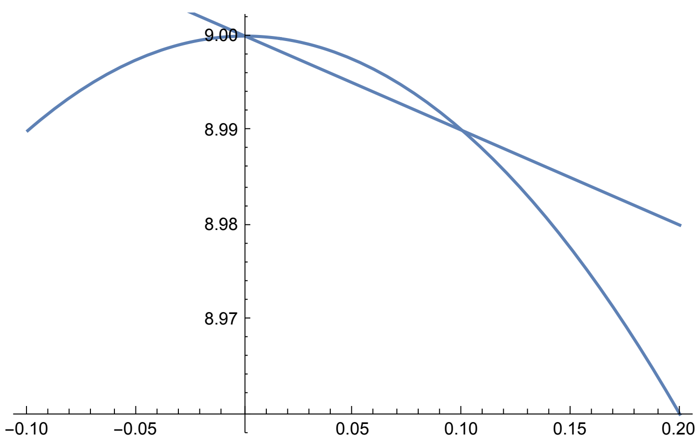



■ [0, 0.1]

In[386]:=
```
x0 = 0.1;
AV[x0]
se1[x_] := f[0] + AV[x0] (x - 0)
p1 = Plot[f[x], {x, -.1, x0 + .1}];
Show[p1, Plot[se1[x], {x, -.1, x0 + .1}]]
```

Out[387]= -0.1

Out[390]=

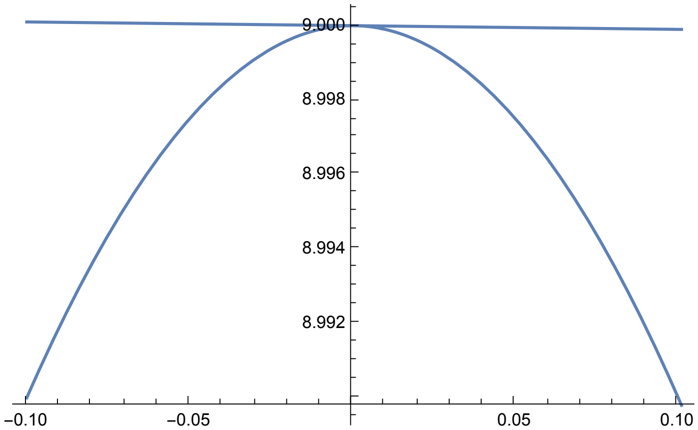



**4.2.** By trying even closer points to $x = 0$, what is your best guess for the instantaneous rate of change for $f$ at $x = 0$?

In[391]:=
```
x0 = 0.001;
AV[x0]
se1[x_] := f[0] + AV[x0] (x - 0)
p1 = Plot[f[x], {x, -.1, x0 + .1}];
Show[p1, Plot[se1[x], {x, -.1, x0 + .1}]]
```

Out[392]= -0.001

Out[395]=




**5.** If $f(x) = \cos(x)$, use the ideas of the previous problems and whatever technology you like to guess the instantaneous rate of change of $f$ at each of the following $x$-values.

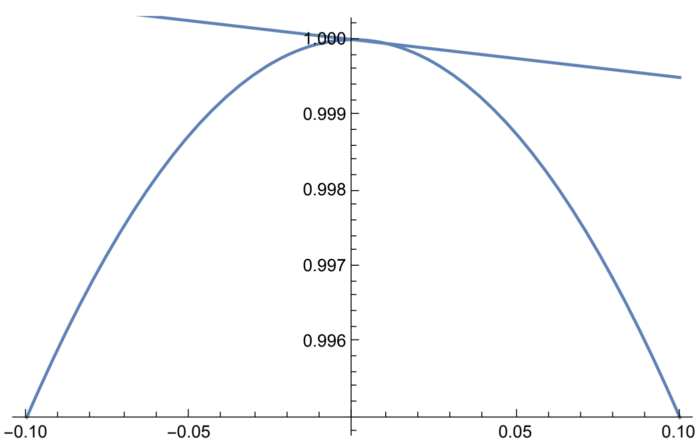
In[396]:=
```
f[x_] := Cos[x]
AV[a_, b_] := (f[a] - f[b]) / (a - b)
```

**5.1.** $x = 0$

In[422]:=
```
a = 0;
x0 = a + 0.01;
AV[x0, a]
se1[x_] := f[a] + AV[x0, a] (x - a)
p1 = Plot[f[x], {x, a - .1, a + .1}];
Show[p1, Plot[se1[x], {x, a - .1, a + .1}]]
```

Out[424]= -0.00499996

Out[427]=

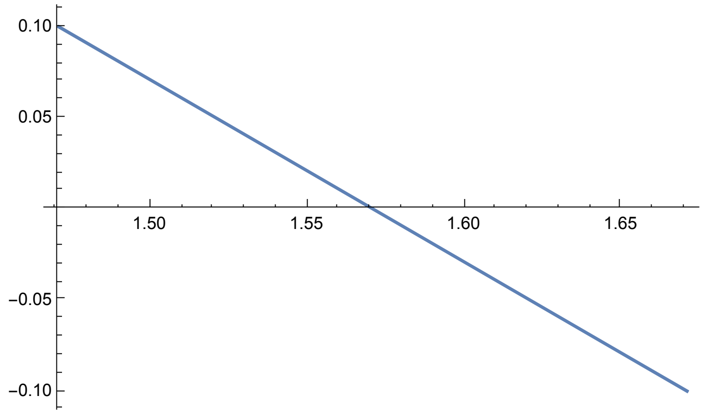



**5.2.** $x = \pi/2$

In[428]:=
```
a = Pi / 2;
x0 = a + 0.01;
AV[x0, a]
se1[x_] := f[a] + AV[x0, a] (x - a)
p1 = Plot[f[x], {x, a - .1, a + .1}];
Show[p1, Plot[se1[x], {x, a - .1, a + .1}]]
```

Out[430]= -0.999983

Out[433]=

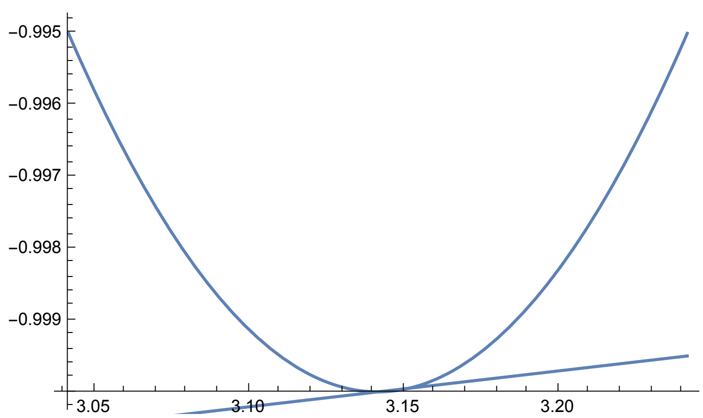



**5.3.** $x = \pi$

In[434]:=
```
a = Pi;
x0 = a + 0.01;
AV[x0, a]
se1[x_] := f[a] + AV[x0, a] (x - a)
p1 = Plot[f[x], {x, a - .1, a + .1}];
Show[p1, Plot[se1[x], {x, a - .1, a + .1}]]
```

Out[436]= 0.00499996

Out[439]=




**5.4.** $x = 3\pi/2$

In[440]:=
```
a = 3 Pi / 2;
x0 = a + 0.01;
AV[x0, a]
se1[x_] := f[a] + AV[x0, a] (x - a)
p1 = Plot[f[x], {x, a - .1, a + .1}];
Show[p1, Plot[se1[x], {x, a - .1, a + .1}]]
```

Out[442]= 0.999983

Out[445]=