MAT128 Final Prep

Problem 1: Use the limit definition of the derivative to find $f'(x)$, when $f(x) = 2x^2 + 1$.

Problem 2: Use standard differentiation rules to calculate the derivative of $f(x) = \dfrac{\sqrt{x}\sin(x^2)}{x+1}$.

Justify each step.

Problem 3: Study and graph the function $f(x) = \dfrac{6x^2}{x^2+1}$. Point out the usual important features of the function.

a. What kind of function is f, and where is it defined?

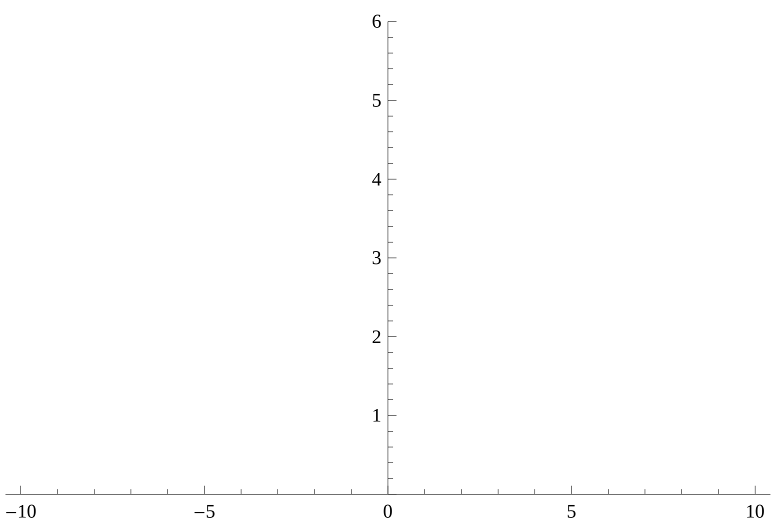
b. What are the function's special properties?

c. Find critical points of f.

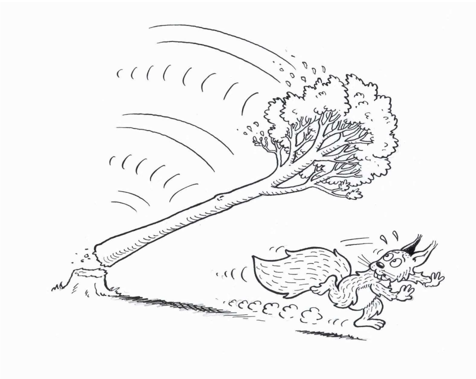
d. Find and classify extrema of f.

e. Find asymptotes of f.

f. Find inflection points of f.



Problem 4: A straight, 100-foot tall tree is falling, cut cleanly at ground level. Something like this figure (but maybe you'd like to clean it up a little to the right of the figure):



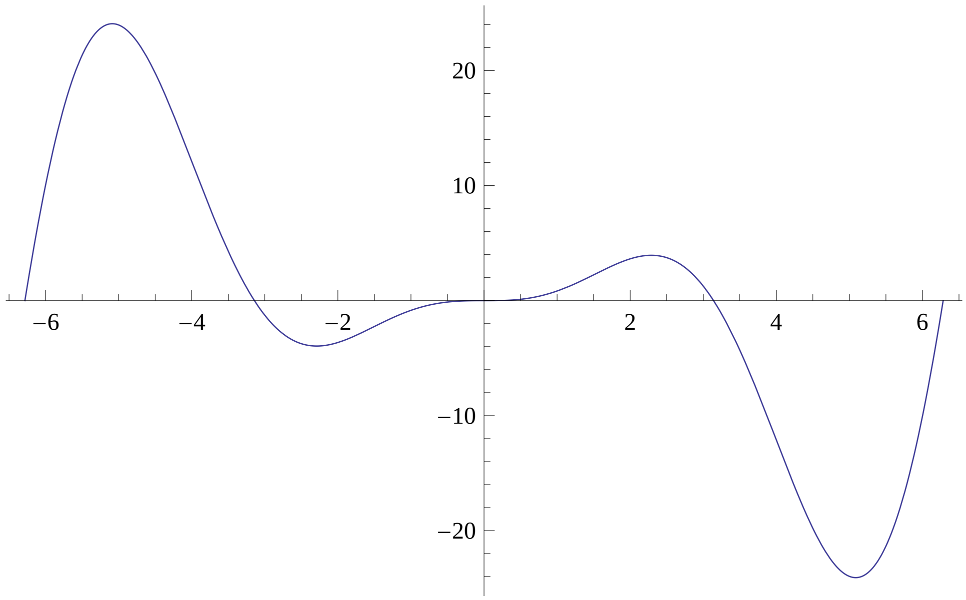
As the tree falls, the angle it makes with the ground changes from 90^o ($\frac{\pi}{2}$ radians) to 0^o. At the moment when the angle is 45^o ($\frac{\pi}{4}$), the angle is changing at a rate of $\frac{\pi}{6}$ radians per second. How fast is the treetop's **height** (from tip straight down to the ground) changing at that moment?

Problem 5: Find the limits:

a. $\displaystyle\lim_{x\to 1} \frac{\arctan(x-1)}{x-1}$

b. $\displaystyle\lim_{x\to 1^+} [(x-1)\ln(x-1)]$

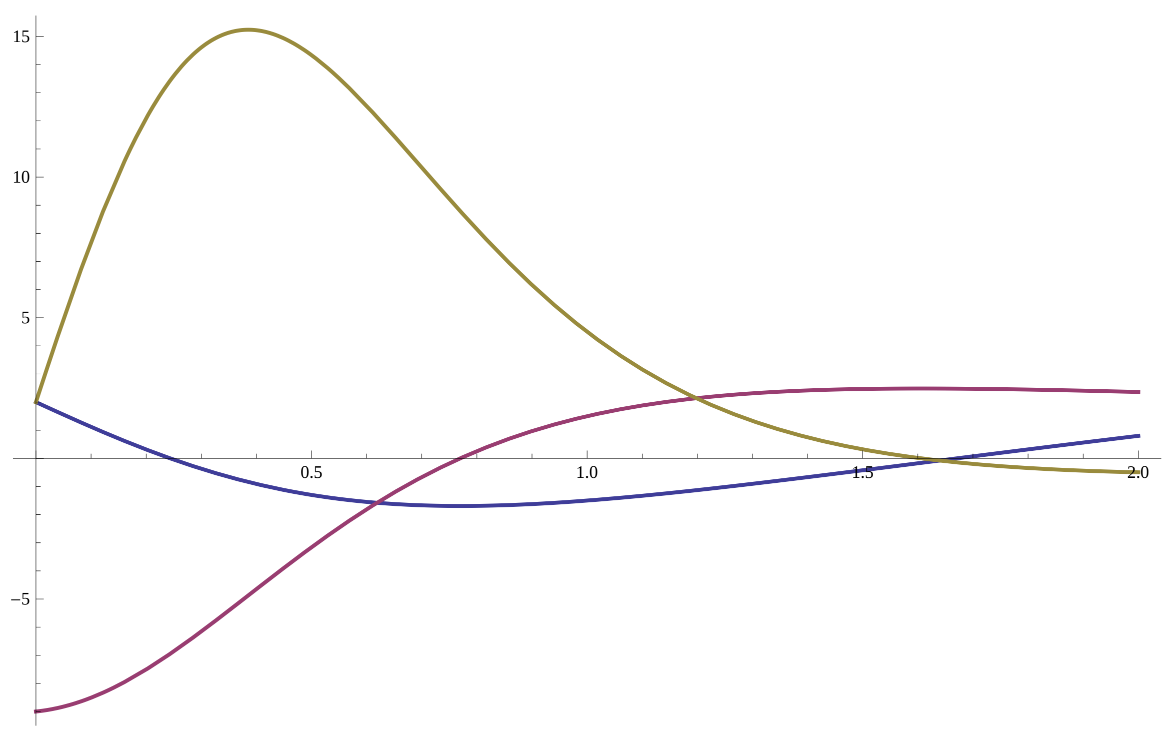
Problem 6: Let $f(x) = x^2 \sin(x)$, graphed below:



a. Sketch the graph of the derivative $f'(x)$ on the same axes.

b. Find the equation of the tangent line to the curve at $x = -\pi$, and add it to your graph.

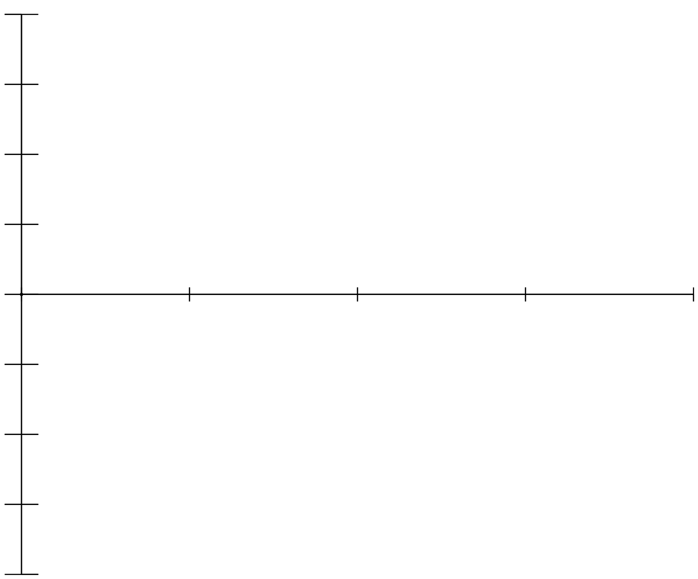
Problem 7: With reasons, determine which of the three graphs in the figure is the function, which its derivative, and which its second derivative: Clearly indicate which is which!



Problem 8: Given the following data (height in meters, time in seconds):

t	0	1	2	3	4	
$h(t)$	-2	1	3	4	3	
average rate of change						

a. Graph the data from the table on the axes below (label!), and use the data to estimate the **average rate of change** over each second. Add your estimates to the table above (where each answer in the box represents the average rate of change over one of the four seconds).



b. Use the data in the table above and an appropriate secant line to estimate the time rate of change of h at $t = 2$. Add your secant and tangent lines to the graph.

Problem 9: Let f be a continuous function on a closed and bounded interval $[a, b]$.

Explain what you can legitimately conclude from each of the following (consider each of these by itself, without regard for the others):

a. The extreme value theorem

b. f is differentiable, and the derivative of f changes sign at $x = c$ from positive to negative.

c. f is twice-differentiable, and the second derivative of f changes sign at $x = c$.

Problem 10: You have 1000 feet of fencing, and want to make two rectangle pens (adjacent to each other and equal in size – think of it as a rectangle subdivided in two). Find the optimal dimensions to maximize the area provided.