

MAT128 Test 3 (Spring 2022):

Name:

Directions: Each problem is worth 10 points. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

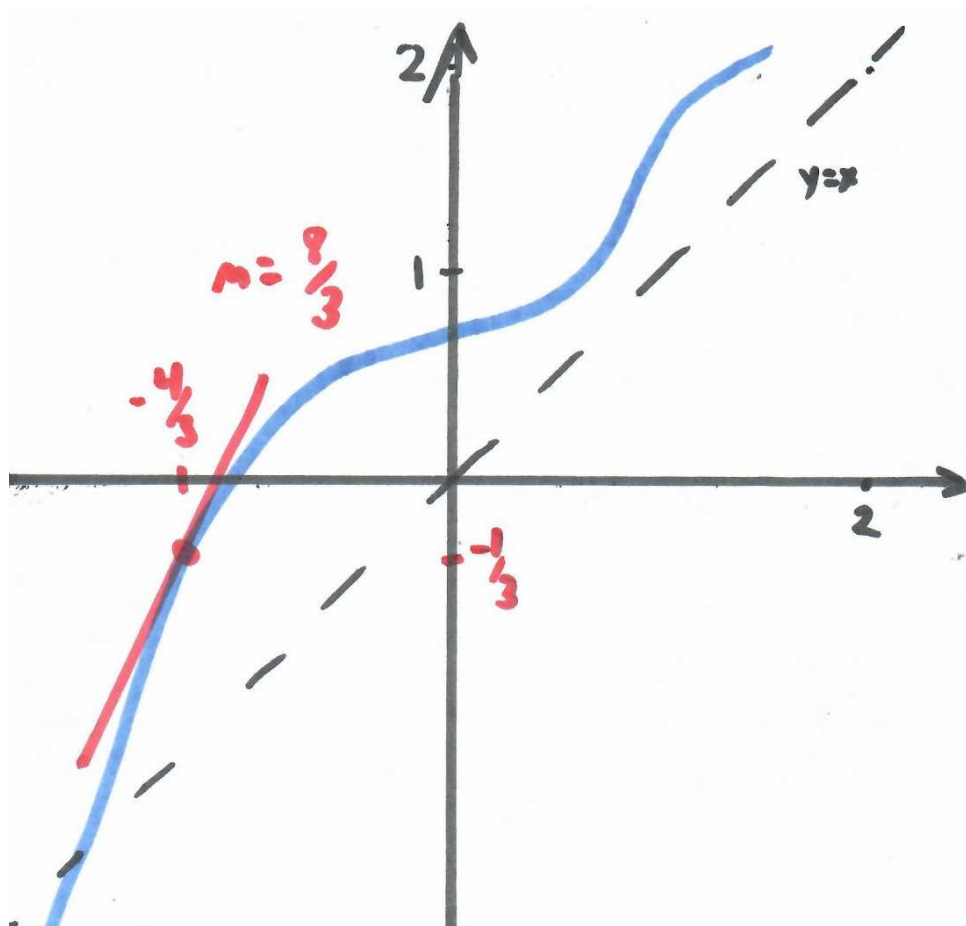
Problem 1: Illustrate the chain rule by differentiating the following functions. Show your choices for the compositions; show all your work.

a. $h(x) = \sin(x^2 - 2x + 1)$

b. $h(x) = (e^x + \cos(x))^3$

Problem 2: Inverse functions:

- a. (4 pts) Draw (carefully!) the inverse function for the function f whose graph is given here:



- b. (2 pts) Also draw (carefully!) the inverse of the tangent line shown in the figure.

- c. (4 pts) Give the equations of both tangent lines.

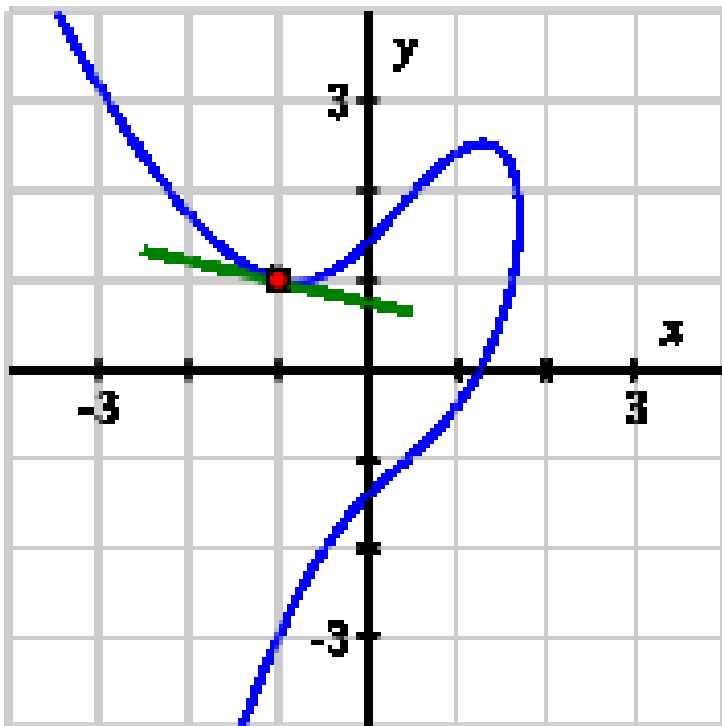
i. The original tangent line:

ii. The tangent line for the inverse:

Problem 3: For the curve given implicitly by

$$x^3 + y^2 - 2xy = 2,$$

shown below,



a. (6 pts) compute $y'(x)$ by implicit differentiation.

b. (4 pts) find the slope of the tangent line at $(-1,1)$, and write the equation of the tangent line shown.

Problem 4: Calculate the following limits (and show/explain your method!). Indicate clearly your final answers for each.

a. $\lim_{x \rightarrow 0} \frac{\tan(x)}{x}$

b. $\lim_{x \rightarrow 1} \frac{(x-1)^2}{(\ln((x-1)^2))}$

c. $\lim_{x \rightarrow \infty} \frac{\arctan(x)}{x}$

d. $\lim_{x \rightarrow 0^+} x \ln(x + 1)$

Problem 5: Consider the function $f(x) = \frac{x-1}{x^2+1}$.

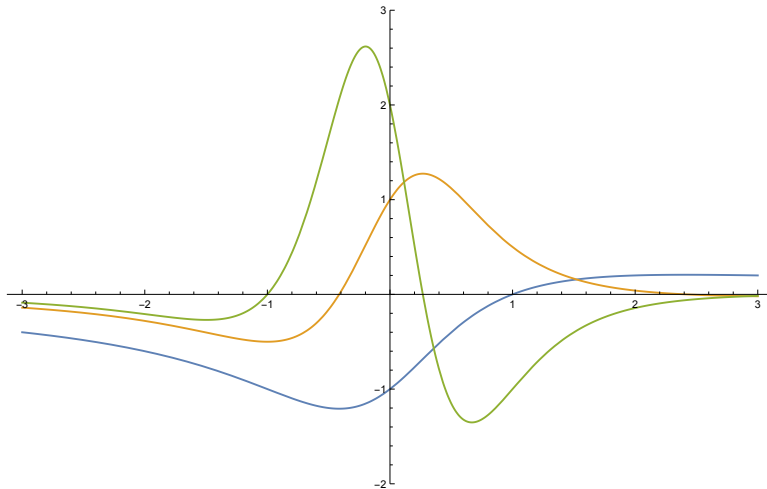
Its first derivative is

$$f'(x) = \frac{-x^2 + 2x + 1}{(x^2 + 1)^2}$$

and the second derivative is

$$f''(x) = \frac{2(x^3 - 3x^2 - 3x + 1)}{(x^2 + 1)^3}$$

The three functions are plotted here:



- (3 pts) Identify which function is which (with reasons) on the graph.
- (5 pts) Using the table below, indicate
 - the zeros and signs of the functions f , f' , and f'' . (If you can't find the **actual** values of the zeros, you can see and name them on the graph.)
 - Asymptote information.

x	
$f(x)$	
$f'(x)$	
$f''(x)$	

- (2 pts) To the right of the graph above, explain how the information in the table is consistent with the graph of f . You might annotate the graphs.

Problem 6: Let $h(x) = \frac{10(x-2)^{2/3}}{x}$. A computer algebra system gives the derivative as

$$h'(x) = \frac{10(6-x)}{3\sqrt[3]{x^2(x-2)}}.$$

a. (2 pts) What are the domains of

i. $h(x)$:

ii. $h'(x)$:

b. (4 pts) Find all the critical numbers of $h(x)$, and for each critical number determine if it is a local maximum, a local minimum, or neither.

c. (4 pts) What are the intervals of increase and the intervals of decrease of $h(x)$?