Exam 3 Prep

MAT128, Spring, 2022

April 18, 2022

Some sample problems over six sections:

- Section 2.5: The chain rule
- Section 2.6: Derivatives of Inverse Functions
- Section 2.7: Derivatives of functions given implicitly
- Section 2.8: Using Derivatives to Evaluate Limits
- Curve sketching
- Section 3.1: Using derivatives to identify extreme values

Since I decided to write this like an exam, I won't have lots of problems for you: but you'll see the style of question I **might** ask.

Problem 1: Illustrate the chain rule by differentiating the following functions:

a. $\ln(x^2 + 1)$

b. $(\ln(x))^2 + 1$

Problem 2: Inverse functions: In our class we chose to restrict cosine to the interval $[0, \pi]$ before creating its inverse.

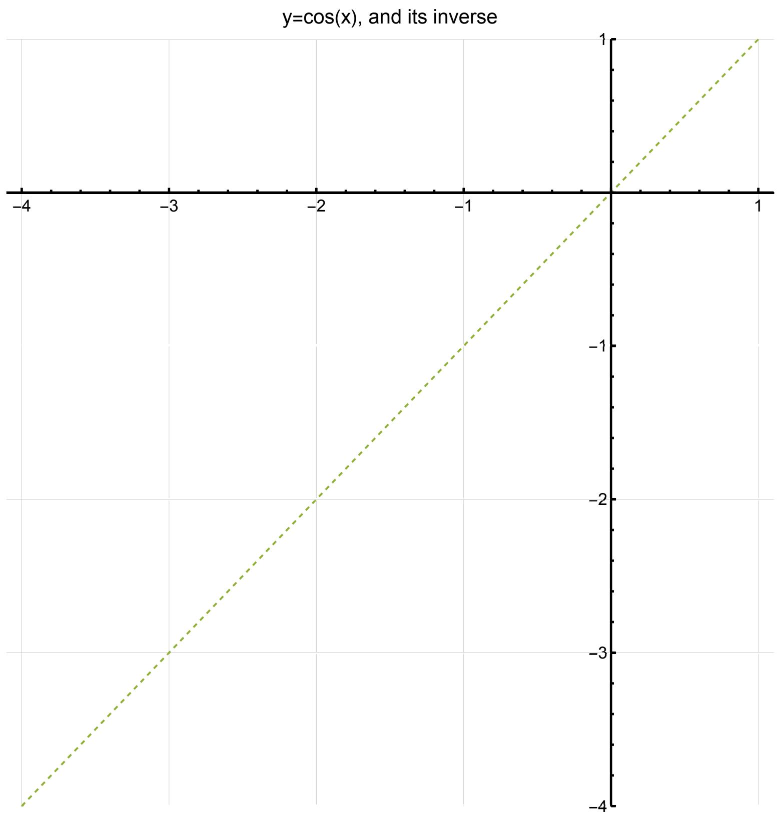
Suppose we had chosen to define the inverse cosine function by restricting the domain of the cosine function to the interval $[-\pi, 0]$.

a. Draw cosine on this interval, and the inverse cosine based on this interval.

b. When we used the interval $[0, \pi]$ for the restriction in class, we calculated the derivative of the inverse cosine as

$$arccos'(x) = \frac{-1}{\sqrt{1 - x^2}}$$

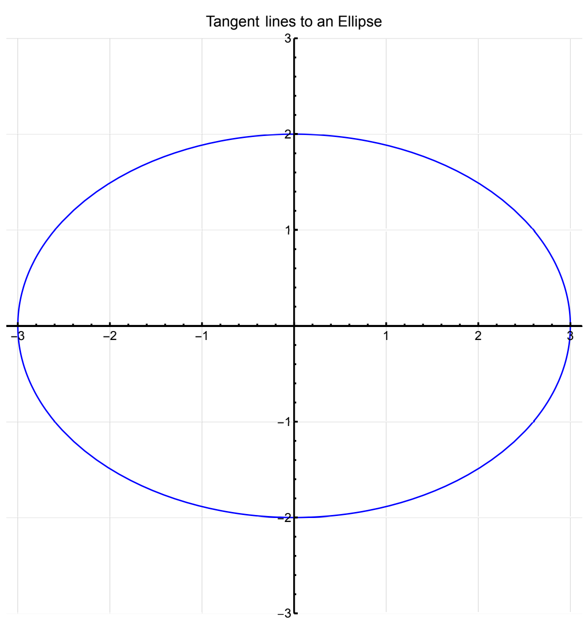
How does this derivative change because we changed the domain of cosine?



Problem 3: Given the equation of the ellipse

$$4x^2 + 9y^2 = 36$$

Draw and find the equations of the **two** lines tangent to the curve when $x = 1$.



Problem 4: Calculate the following limits (and show/explain your method!):

a. $\lim_{x\to 0} \frac{\cos(x)}{x^2}$

b. $\lim_{x\to 0} \frac{\tan(x)}{x}$

c. $\lim_{x\to 0^+} x\ln(x)$

Problem 5: Consider the function

$$f(x) = \frac{(x-1)(x+1)}{x^2+1}$$

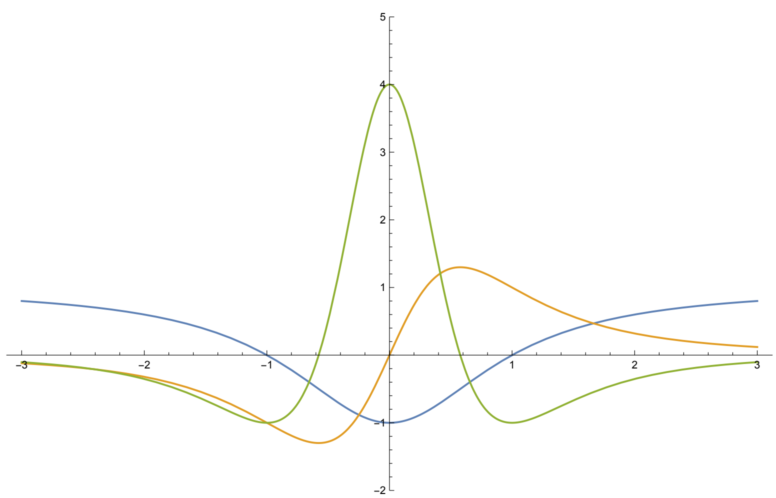
The first derivative is

$$f'(x) = \frac{4x}{(x^2+1)^2}$$

and the second derivative is

$$f''(x) = \frac{4-12x^2}{(x^2+1)^3}$$

The three functions are plotted here:



a. Identify which function is which (with reasons) on the graph.

b. Create a table of the zeros and signs of the functions f, f', and f'', and explain how the information in the table is consistent with the graph of f.

Problem 6: In the previous problem, illustrate the first and second derivative tests, and how they are consistent with any local extrema in the graph of the function f.