Activity 1.8.3. This activity concerns a function $f(x)$ about which the following information is known:

- f is a differentiable function defined at every real number x
- $f(2) = -1$
- $y = f'(x)$ has its graph given in Figure 1.8.6

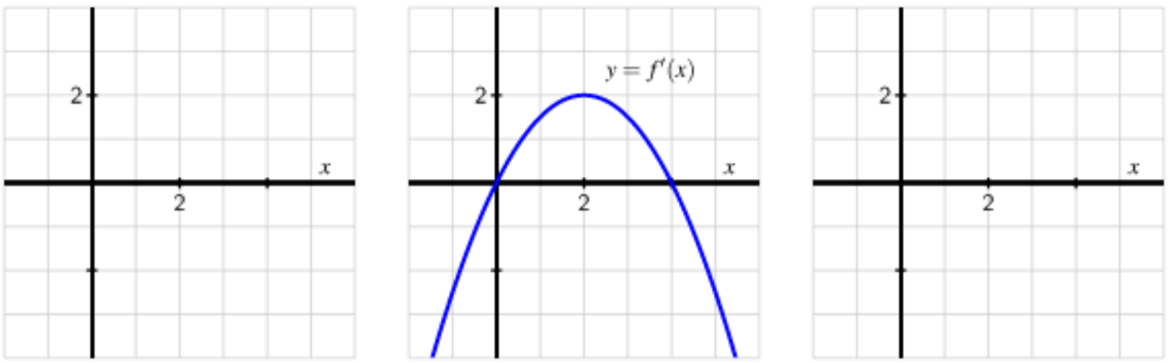

Figure 1.8.6. At center, a graph of $y = f'(x)$; at left, axes for plotting $y = f(x)$; at right, axes for plotting $y = f''(x)$.

Your task is to determine as much information as possible about f (especially near the value $a = 2$) by responding to the questions below.

a. Find a formula for the tangent line approximation, $L(x)$, to f at the point $(2, -1)$.

b. Use the tangent line approximation to estimate the value of $f(2.07)$. Show your work carefully and clearly.

c. Sketch a graph of $y = f''(x)$ on the righthand grid in Figure 1.8.6; label it appropriately.

d. Is the slope of the tangent line to $y = f(x)$ increasing, decreasing, or neither when $x = 2$? Explain.

e. Sketch a possible graph of $y = f(x)$ near $x = 2$ on the lefthand grid in Figure 1.8.6. Include a sketch of $y = L(x)$ (found in part (a)). Explain how you know the graph of $y = f(x)$ looks like you have drawn it.

f. Does your estimate in (b) over- or under-estimate the true value of $f(2.07)$? Why?

6. A potato is placed in an oven, and the potato's temperature F (in degrees Fahrenheit) at various points in time is taken and recorded in the following table. Time t is measured in minutes.

Table 1.8.7. *Temperature data for the potato.*

t	$F(t)$
0	70
15	180.5
30	251
45	296
60	324.5
75	342.8
90	354.5

a. Use a central difference to estimate $F'(60)$. Use this estimate as needed in subsequent questions.

b. Find the local linearization $y = L(t)$ to the function $y = F(t)$ at the point where $a = 60$.

c. Determine an estimate for $F(63)$ by employing the local linearization.

d. Do you think your estimate in (c) is too large or too small? Why?

7. An object moving along a straight line path has a differentiable position function $y = s(t)$; $s(t)$ measures the object's position relative to the origin at time t. It is known that at time $t = 9$ seconds, the object's position is $s(9) = 4$ feet (i.e., 4 feet to the right of the origin). Furthermore, the object's instantaneous velocity at $t = 9$ is -1.2 feet per second, and its acceleration at the same instant is 0.08 feet per second per second.

a. Use local linearity to estimate the position of the object at $t = 9.34$.

b. Is your estimate likely too large or too small? Why?

c. In everyday language, describe the behavior of the moving object at $t = 9$. Is it moving toward the origin or away from it? Is its velocity increasing or decreasing?