Power, Quotient, Exponential, and Sine/Cosine Worksheet

1. Let $f(x) = \sqrt{x}$.

1.1. Get the linear approximation to $f(x)$ near $x = 1$. Call it $L(x)$.

1.2. Make a table of values for your linear approximations, the exact values, and the absolute errors.

x	−0.2	0.1	0.4	0.7	1.0	1.3	1.6	1.9	2.2	2.5
$L(x)$										
$f(x)$										
$\lvert f(x) - L(x) \rvert$										

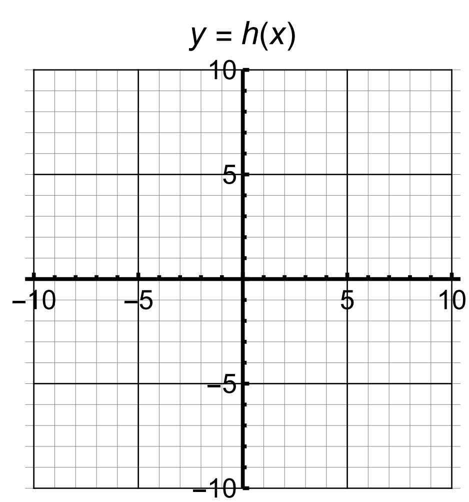
1.3. For which values of x does your linear approximation fall within 0.1 in absolute error of the exact value?

2. Let $h(x) = \frac{5x+5}{x^2+x+1}$.

2.1. Is $h'(5)$ negative, 0, or positive? Write the equation of the tangent line at $x = 5$, **in point-slope form** (do not "simplify").

2.2. Determine all the values of x where the $y = h(x)$ will have horizontal tangents by solving $h'(x) = 0$.

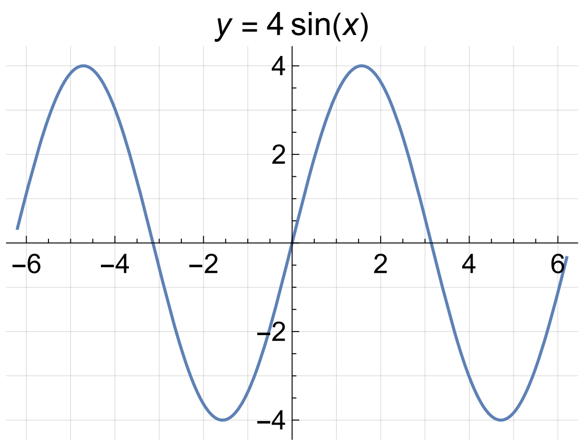
2.3. Graph $y = h(x)$ below. Put points on the graph where $x = 5$ and where you found horizontal tangents. Does the graph match what you expected in the questions above?



3. Consider the function $f(x) = 4\sin(x)$. Its graph is shown below.

3.1. Draw the tangent line at each of the give x-values and estimate its slope. (Get the intermediate values by symmetry!)

x	−6	□	−4	□	−2	□	0	1	□	3	□
slope of tangent line											



3.2. Compute the derivative $f'(x)$ and use it to make the table of values.

x	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
$f'(x)$													

3.3. Did the two tables match?

4. Let $h(x) = \frac{\sin(x)}{1+x^2}$.

4.1. Determine the linear approximation $L(x)$ to $h(x)$ around $x = 0$.

4.2. Use your linear approximation to estimate $h(-0.2)$.

4.3. Plot $L(x)$ and $h(x)$ in the vicinity of $x = 0$, and comment on how well it did at estimating $h(-0.2)$.

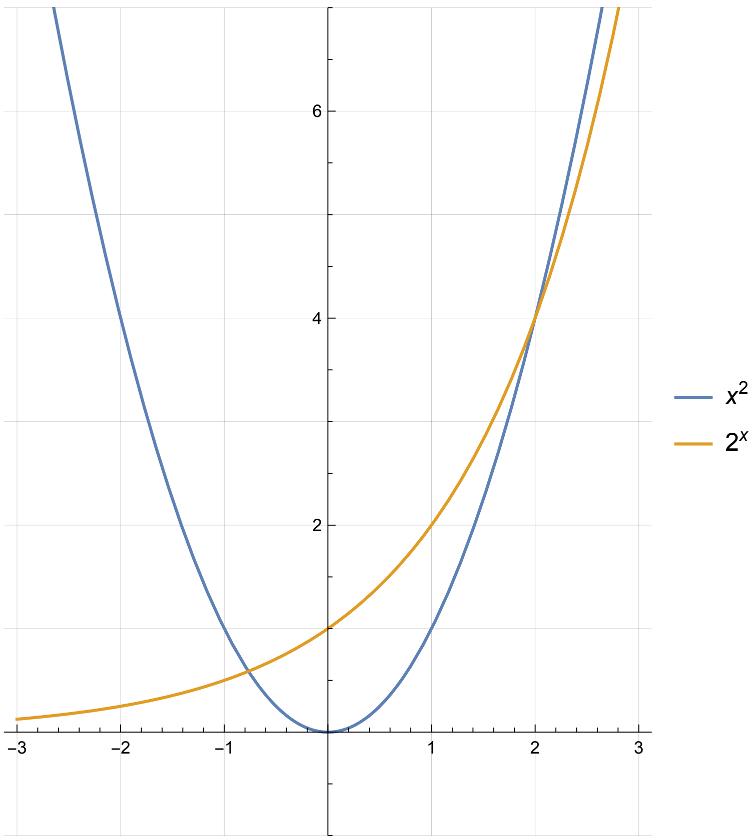
Derivative of exponentials

Comparison

Power function: $f(x) = x^2$

Exponential function: $g(x) = 2^x$

Out[ ]=



Derivatives of exponential functions.

- Let $f(x) = 2^x$. Then

$$f'(x) = \lim_{h\to 0} \frac{2^{x+h}-2^x}{h}$$

$$= \lim_{h\to 0} 2^x \frac{(2^h-1)}{h}$$

$$= 2^x \lim_{h\to 0} \frac{2^h-1}{h}$$

h	0.1	0.01	0.001	0.0001	0.00001
$\frac{2^h-1}{h}$	0.717735	0.695555	0.693387	0.693171	0.69315

$f'(x) = 0.693\,(3^x) = 0.693\,f(x)$

- Let $f(x) = 3^x$. Then

$$f'(x) = \lim_{h \to 0} \frac{3^{x+h} - 3^x}{h}$$
$$= \lim_{h \to 0} 3^x \frac{(3^h - 1)}{h}$$
$$= 3^x \lim_{h \to 0} \frac{3^h - 1}{h}$$

h	0.1	0.01	0.001	0.0001	0.00001
$\frac{3^h-1}{h}$	1.16123	1.10467	1.09922	1.09867	1.09862

$f'(x) = 1.0986\,(3^x) = 1.0986\, f(x)$

5. Natural base:

We might guess that somewhere between 2 and 3, there is a base such that $f'(x) = f(x)$. In fact, there is a number e with $2 < e < 3$ such that if $f(x) = e^x$, then $f'(x) = 1 \cdot e^x = e^x$.

Euler's number is $e = 2.71828\ldots$ Complete a table like those above, and verify that $f'(x) = f(x)$.

h	0.1	0.01	0.001	0.0001	0.00001
$\frac{e^h-1}{h}$					

Questions

- 5.1. What is the slope of the tangent line to $y = e^x$ when $x = -1$? When $x = 1$?

- 5.2. Find an equation for the tangent line to $y = x^3 - e^x + 2$ when $x = 0$.