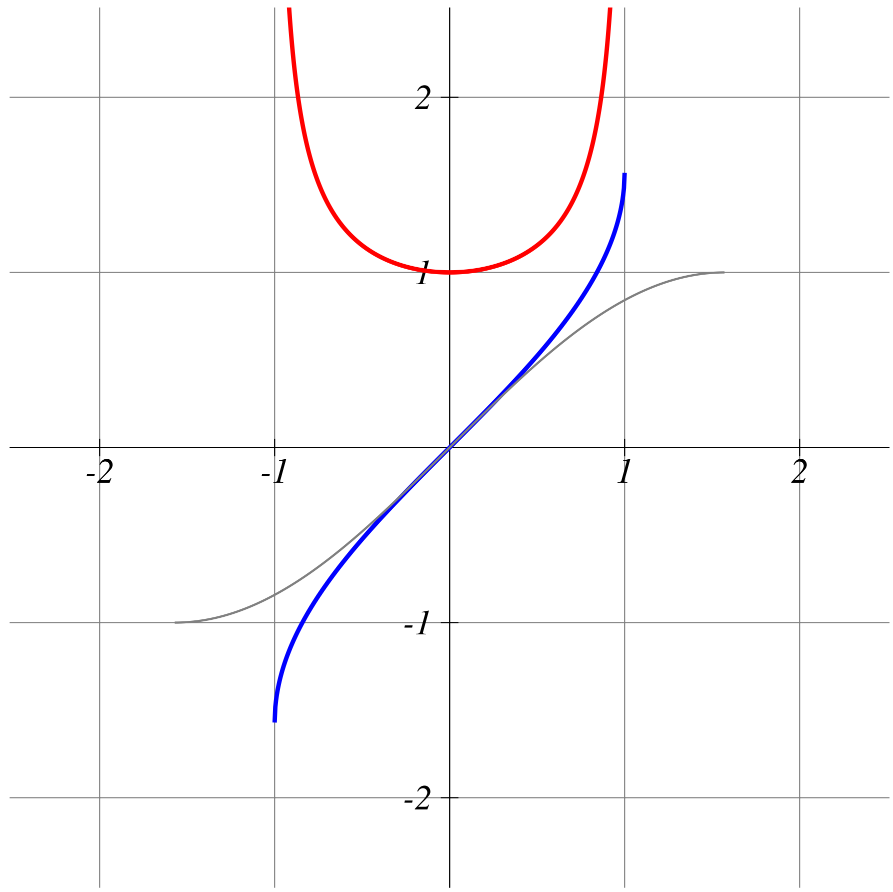
$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

where you will notice that I chose the positive "branch". Why?

Look at sine on this interval. If it is increasing everywhere, its inverse must be increasing everywhere: and what does that say about its derivative?



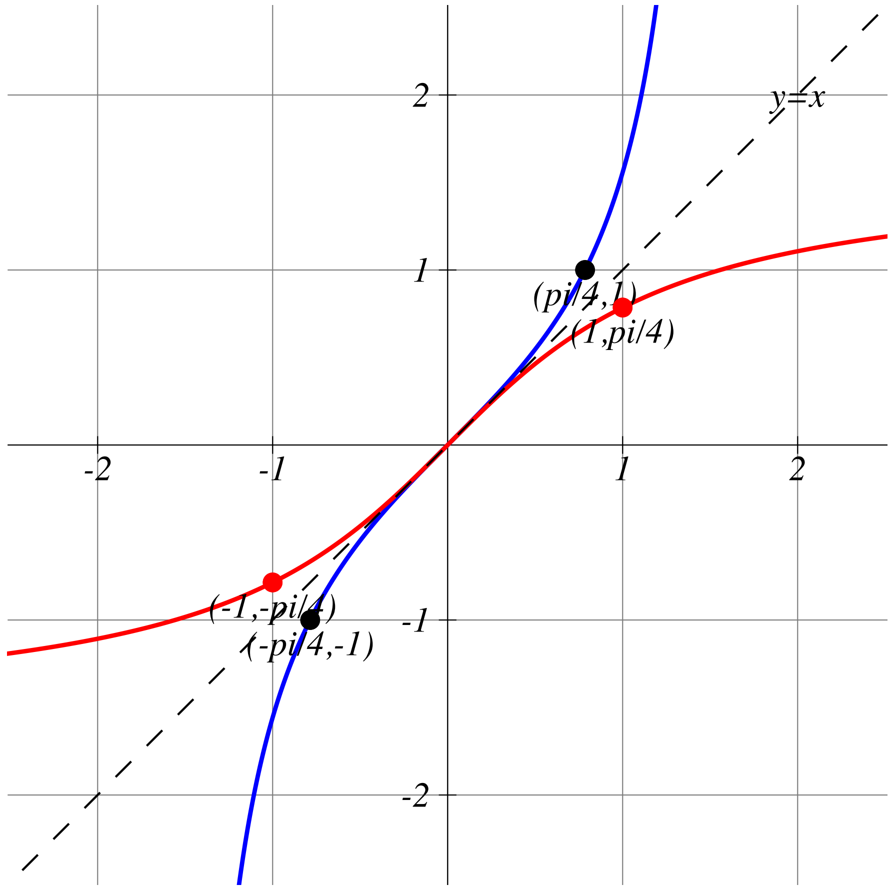
- Your turn!

  There are two other functions which deserve our attention. We call their inverses "arctan" and "arccos".

  Do exactly the same thing that we've just done in these three examples, but with two other important functions:

  1. $f(x) = \tan(x)$ -- use restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

     - Just as before, but once again we have to restrict the domain: tangent is not invertible. We have a choice, but it seems like the best place to think of tangent as invertible is on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:



- Now how about its derivative? For $f(x) = \tan(x)$, $f'(x) = \sec^2(x)$. But this time it will be a little more unpleasant to compute $f'(f^{-1}(x))$. There's a trick you know, however: we can rewrite secant in terms of tangent, using our most important trig identity:

  $$\sin(x)^2 + \cos(x)^2 = 1,$$

  Multiply through (both sides) by $\frac{1}{\cos^2(x)}$ to get

  $$\tan^2(x) + 1 = \sec^2(x),$$

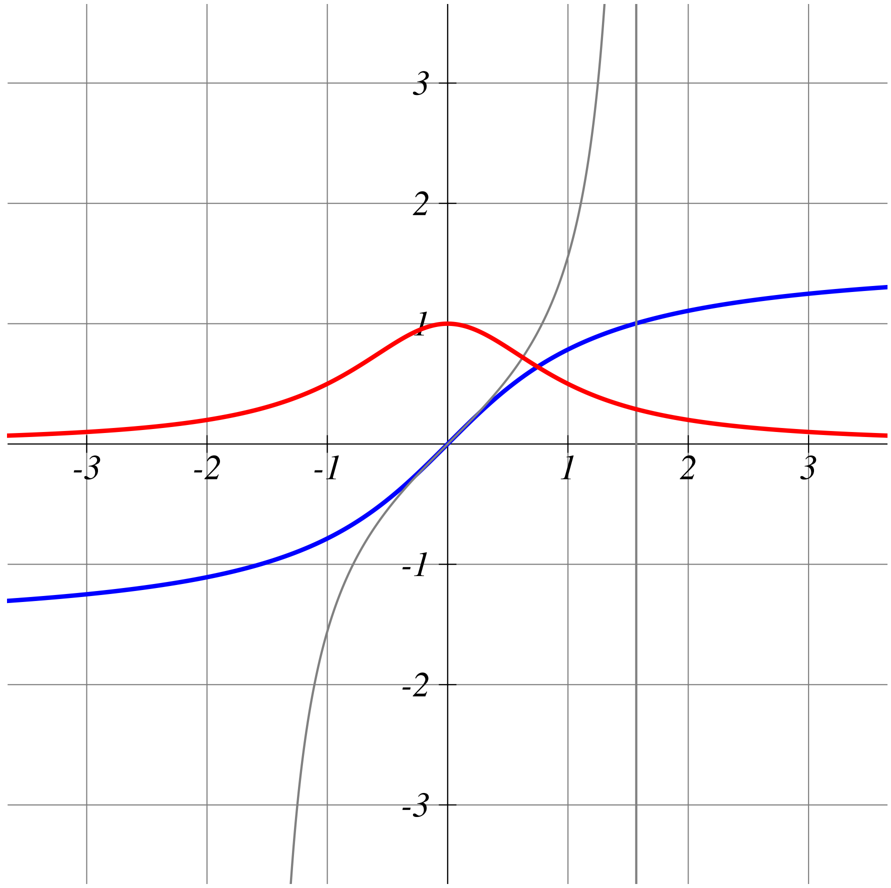
  Hence

  $$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{1 + \tan^2(\arctan(x))}$$

  becomes

  $$\frac{d}{dx}(\arctan(x)) = \frac{1}{1 + x^2}$$

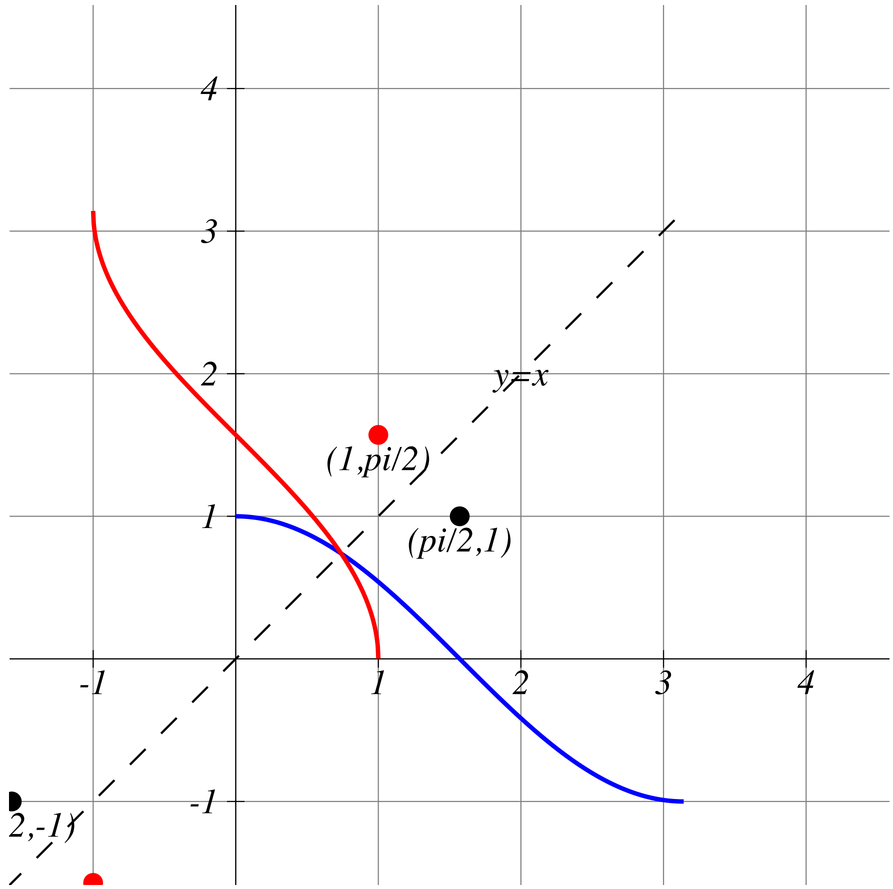
  Look at tangent on this interval. If it is increasing everywhere, its inverse must be increasing everywhere: and what does that say about its derivative? Positive everywhere:



Notice the symmetry: arctan is odd, and of course its derivative is even.

- $f(x) = \cos(x)$ -- use restricted domain $[0, \pi]$: the inverse function of $f(x) = \cos(x)$: we'll call it $f^{-1}(x) = \arccos(x)$.

  - Just as before, but once again we have to restrict the domain: cosine is not invertible. We have a choice, but it seems like the best place to think of cosine as invertible is on the interval $[0, \pi]$:



- Now how about its derivative? For $f(x) = \cos(x)$, $f'(x) = -\sin(x)$ -- still easy! But this time it will be a little more unpleasant to compute $f'(f^{-1}(x))$. Rewrite sine in terms of cosine, using the Pythagorean identity: so

$$\sin(x) = \pm\sqrt{1-\cos(x)^2},$$

Hence

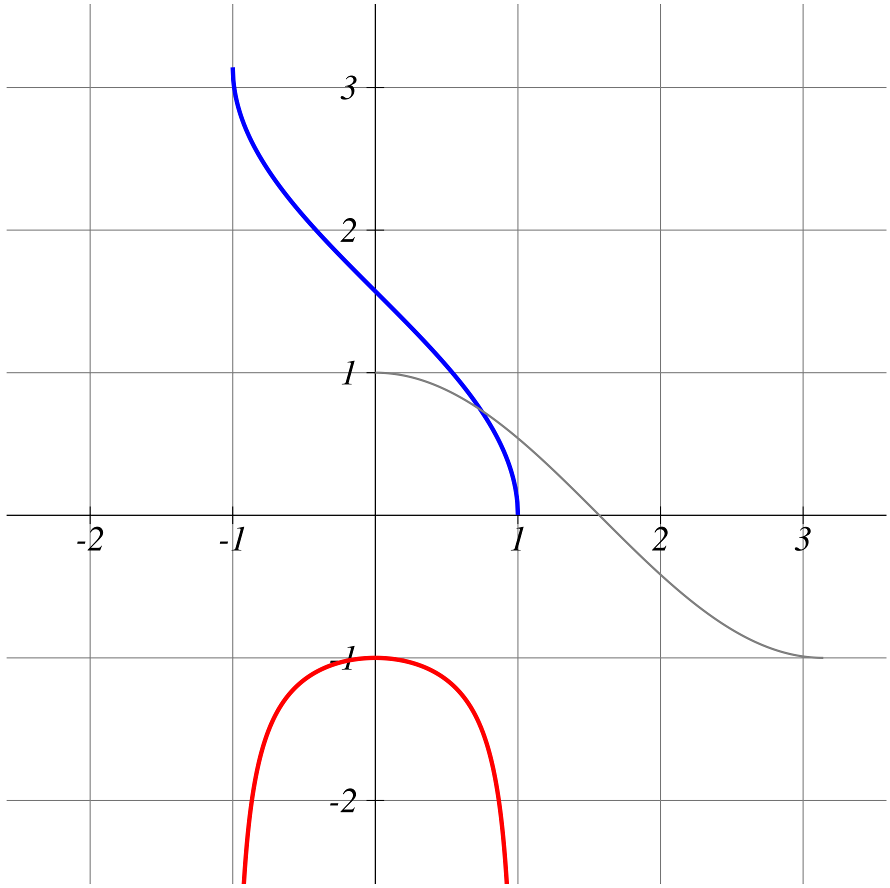
$$\frac{d}{dx}\left(f^{-1}(x)\right) = -\frac{1}{\pm\sqrt{1-\cos(\arccos(x))^2}}$$

becomes

$$\frac{d}{dx}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$$

where you will notice that I chose the negative "branch":

Look at cosine on this interval. If it is decreasing everywhere, its inverse must be decreasing everywhere: and what does that say about its derivative? The derivative is negative:



- **Links**:
    - A Chain Rule tutorial
    - Our free, on-line textbook, called Active Calculus.

Website maintained by Andy Long. Comments appreciated.
Updated on 04/11/2022 01:01:11