Curve graphing worksheet (End of Chapter 2!)

Study and graph each function f(x) as we did in class. Name:

1. Let $f(x) = e^{\sin(x)} + 1$.

In[25]:=
```
f[x_] := E^Sin[x] + 1
```

1.1. Study the function’s form, symmetries, periodicity, domain and range, etc.

This function is periodic of period 2pi. We will study it on the interval [0,2pi]. Since sine oscillates between -1 and 1, this function will oscillate between e^{-1}+1 and e^1+1.

Since both e^x and sin(x) are very smooth functions, their composition will be smooth as well. The domain of both of these functions is all real numbers.

In[26]:=
```
f'[x]
```

Out[26]= $e^{\text{Sin}[x]}\ \text{Cos}[x]$

1.2. Study the derivative

$f'(x) = \cos(x)\, e^{\sin(x)}$

This function is 0 on the interval [0,2pi] at two points: pi/2 and 3pi/2. It starts off positive, then negative, then positive again (its sign is the sign of cos(x), since e^(sin(x)) is always positive).

In[28]:=
```
Simplify[f''[x]]
```

Out[28]= $e^{\text{Sin}[x]}\ (\text{Cos}[x]^2 - \text{Sin}[x])$

1.3. Study the second derivative

$f''(x) = (\cos(x)\,^\wedge 2 - \sin(x))\, e^{\sin(x)}$

Again, the sign is that of what multiplies $e^{\sin(x)}$, since that term is always positive.

We can use Pythagoras to help us study the sign of that term:
cos (x)^2-sin(x) = 1-sin(x)-sin(x)^2

so we find y such that 1-y-y^2=0, which is when y = -(1 + sqrt(5))/2 and y = -(1 - sqrt(5))/2, and then we solve for when sin(x)=y (only works for the latter): x→0.666239, x→2.47535

In[30]:= `Solve[f''[x] == 0.0, x]`

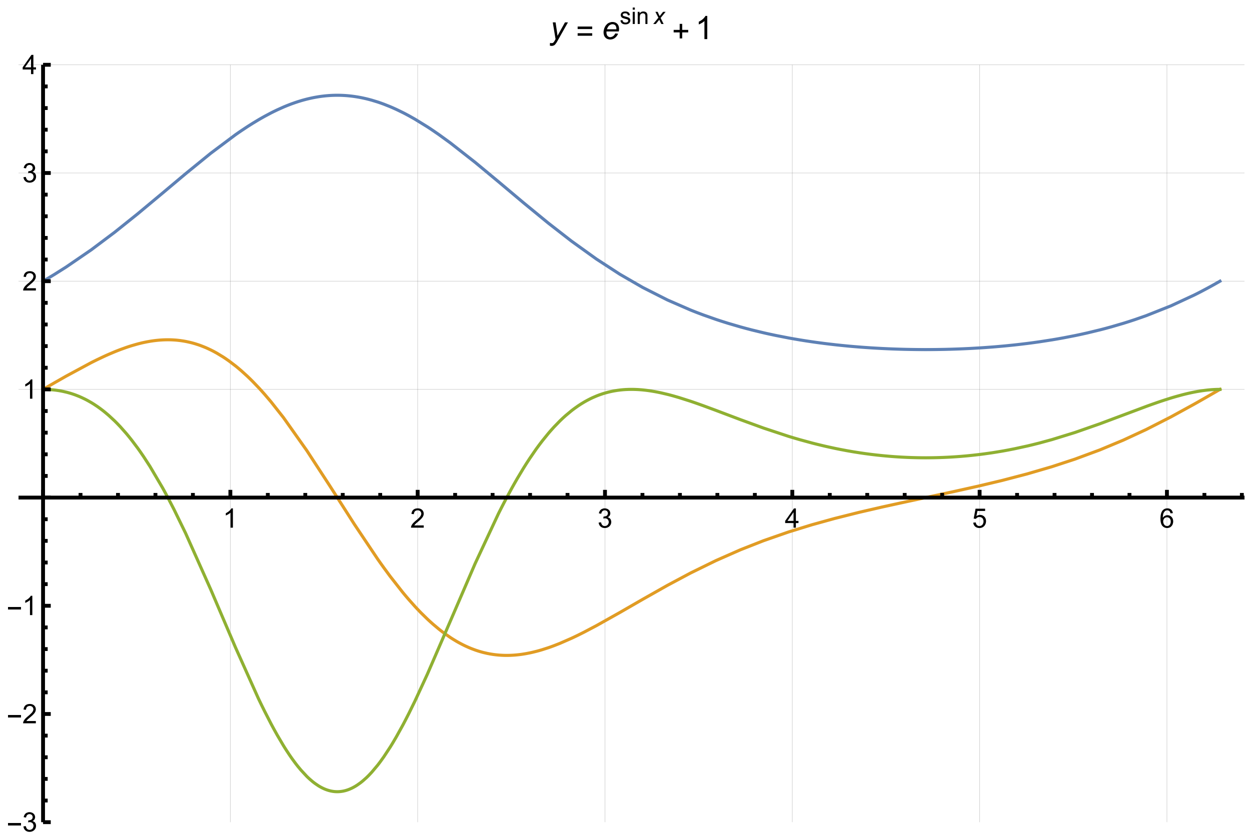
Out[30]= {{x → -1.5708 - 1.06128 ⅈ}, {x → -1.5708 + 1.06128 ⅈ}, {x → 0.666239}, {x → 2.47535}}

1.4. Build a table of all the important information.

x	0	□	pi/2	0.666	□	□	2.47	3 pi/2	□	2 pi
$f(x)$	2		e^1 + 1					e^(−1) + 1		2
$f'(x)$	1	+	0	−	−	−	−	0	+	1
$f''(x)$	1	+	+	0	−	−	0	+	+	1

1.5. Plot your function on the grid below. Indicate major points of interest on the graph.

Out[66]=



2. Let $f(x) = \frac{x^2-1}{x^2+1}$.

In[70]:=
```
f[x_] := (x^2 - 1) / (x^2 + 1)
g[x_] := 1 - 2 / (x^2 + 1)
Simplify[f[x]]
Simplify[g[x]]
```

Out[72]= $\frac{-1+x^2}{1+x^2}$

Out[73]= $1-\frac{2}{1+x^2}$

2.1. Study the function's form, symmetries, periodicity, domain and range, etc.

This function is even, since it involves only even powers of x. So we could study it on [0,infinity), for example.

It's a rational function, and it can be re-written as $1-\frac{2}{1+x^2}$. This shows that its horizontal asymptote is 1, and that it is always below 1. So it has to have a minimum at x=0.

In[74]:=
```
g'[x]
```

Out[74]= $\frac{4\,x}{\left(1+x^2\right)^2}$

2.2. Study the derivative

The derivative is 0 at 0, when we have a minimum. Its sign is negative to the left, and positive to the right of zero.

It is also a rational function, which goes to zero as x gets large.

In[76]:=
```
Simplify[g''[x]]
```

Out[76]= $\frac{4-12\,x^2}{\left(1+x^2\right)^3}$

2.3. Study the second derivative

The second derivative is $\frac{4\left(1-3\,x^2\right)}{\left(1+x^2\right)^3}$. This function is zero when x = 2/sqrt(12), or 1/sqrt(3). It is negative when x is large in absolute value, and positive between its two roots.

2.4. Build a table of all the important information.

x	$-\infty$	-1	$\frac{-1}{\sqrt{3}}$	□	0	□	$\frac{1}{\sqrt{3}}$	1	□	∞
$f(x)$	1	0			-1			0		1
$f'(x)$	0	–	–	–	0	+	+	+		0
$f''(x)$	0	–	0	+	4	+	0	–		0

2.5. Plot your function on the grid below. Indicate major points of interest on the graph.

Out[94]=