3.4 and 3.5: Story Problems to help prepare for the final

The skateboard problem from 3.5 is well illustrated with the applet referenced:

Activity 3.5.4. As pictured in the applet at http://gvsu.edu/s/9q, a skateboarder who is 6 feet tall rides under a 15 foot tall lamppost at a constant rate of 3 feet per second. We are interested in understanding how fast his shadow is changing at various points in time.

a. Draw an appropriate right triangle that represents a snapshot in time of the skateboarder, lamppost, and his shadow. Let x denote the horizontal distance from the base of the lamppost to the skateboarder and s represent the length of his shadow. Label these quantities, as well as the skateboarder's height and the lamppost's height on the diagram.

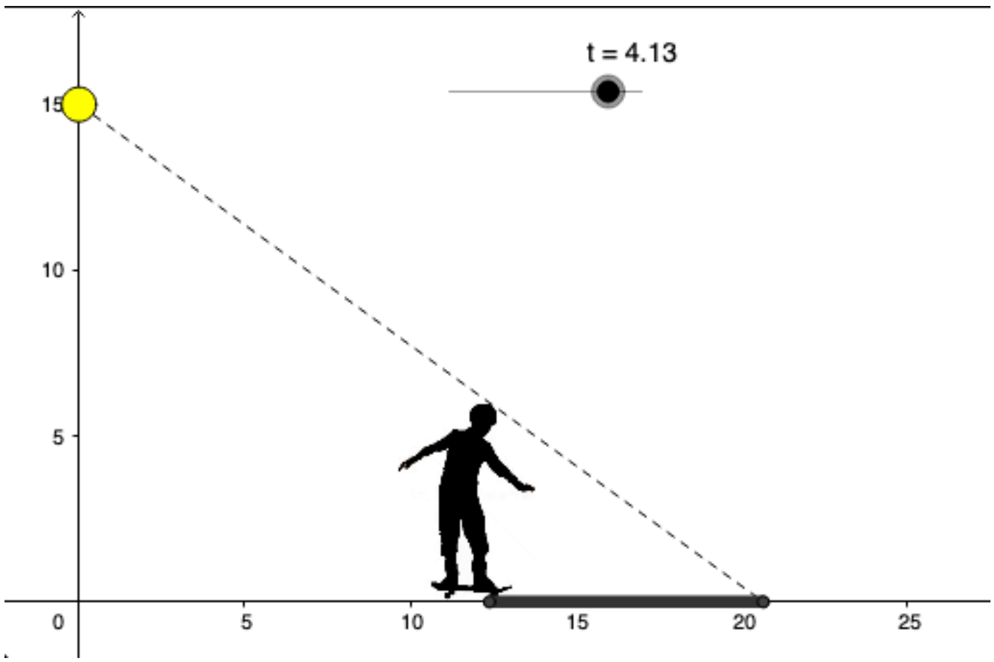
b. Observe that the skateboarder and the lamppost represent parallel line segments in the diagram, and thus similar triangles are present. Use similar triangles to establish an equation that relates x and s.

c. Use your work in (b) to find an equation that relates $\frac{dx}{dt}$ and $\frac{ds}{dt}$.

d. At what rate is the length of the skateboarder's shadow increasing at the instant the skateboarder is 8 feet from the lamppost?

e. As the skateboarder's distance from the lamppost increases, is his shadow's length increasing at an increasing rate, increasing at a decreasing rate, or increasing at a constant rate?

f. Which is moving more rapidly: the skateboarder or the tip of his shadow? Explain, and justify your answer.



3.4.8. Two vertical poles of heights 60 ft and 80 ft stand on level ground, with their bases 100 ft apart. A cable that is stretched from the top of one pole to some point on the ground between the poles, and then to the top of the other pole. What is the minimum possible length of cable required? Justify your answer completely using calculus.

In[214]:=
```
distanceBetween = 100
height1 = 60
height2 = 80
(* x is between 0 and 100 *)
d1[x_] := Sqrt[height1^2 + x^2]
d2[x_] := Sqrt[height2^2 + (100 - x)^2]
length[x_] := d1[x] + d2[x]
length[x]
dldx = Together[D[length[x], x]]
optx = x /. Solve[dldx == 0, x]〚1〛〚1〛
optl = length[optx]
N[%]
Show[
 Plot[length[x], {x, 0, 100}],
 ListPlot[{{optx, optl}}],
 PlotRange → {165, 190}
]
```

Out[214]= 100

Out[215]= 60

Out[216]= 80

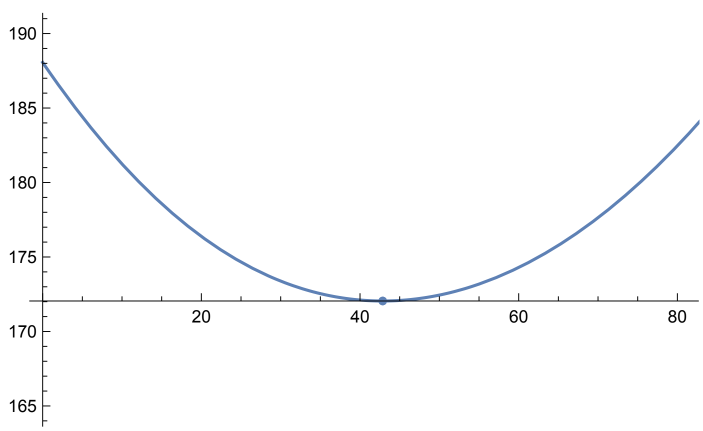
Out[220]= $\sqrt{6400 + (100 - x)^2} + \sqrt{3600 + x^2}$

Out[221]= $\dfrac{-100\sqrt{3600 + x^2} + x\sqrt{3600 + x^2} + x\sqrt{16400 - 200x + x^2}}{\sqrt{3600 + x^2}\sqrt{16400 - 200x + x^2}}$

Out[222]= $\dfrac{300}{7}$

Out[223]= $20\sqrt{74}$

Out[224]= 172.047

Out[225]=


In[210]:=
```
Simplify[(100 - x) ^2 (60 ^2 + x ^2) - x ^2 (80 ^2 + (100 - x) ^2)]
driv[x_] := (100 - x) ^2 (60 ^2 + x ^2) - x ^2 (80 ^2 + (100 - x) ^2)
driv[optx]
```

Out[210]= $-400\left(-90000 + 1800x + 7x^2\right)$

Out[212]= 0

In[213]:=
```
Solve[(100 - x) ^2 (60 ^2 + x ^2) - x ^2 (80 ^2 + (100 - x) ^2) == 0, x]
```

Out[213]= $\left\{\{x \to -300\}, \left\{x \to \frac{300}{7}\right\}\right\}$

In[208]:= drive[x_] := $-100\sqrt{3600 + x^2} + x\sqrt{3600 + x^2} + x\sqrt{16400 - 200x + x^2}$

```
drive[optx]
```

Out[209]= 0