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On the Colouring of Maps

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of geography had come to the front. In sending the forces into the field, the Government, inspired no doubt in a great measure by General Thuillier, had provided, it seemed, ample means for obtaining geographical information which would last for all time, and probably would be the most valuable result of the present expedition. He himself was now in correspondence with General Biddulph, who had already sent home some very valuable reconnaissance maps of the southern part of the country. The General indeed had conducted a reconnoitring force 50 miles to the east of Quetta as far as Amadun, and his sketch map of that region contained entirely new geographical information. Sir Henry had recently received a letter from General Biddulph which was written just as he was starting from the Helmund, and in that letter the General said he would take care to get a sketch map of that region, so that as far as the western part of Afghanistan was concerned a considerable accession to our geographical knowledge might confidently be expected.

General THUILLIER wished to add that two other officers of the Survey Department of India, Captain Holdich, R.E., and Captain Maxwell Campbell, R.E., having received a telegram offering them employment with the army in Afghanistan, had left London at two or three days' notice for that express purpose. Lieutenant E. P. Leach, R.E., had likewise returned to India for a similar purpose.

Sir HENRY RAWLINSON, in reply to a question put by Mr. J. L. HADDAN respecting the height of Lake Sistan above the sea-level, said he believed it was between 800 and 1000 feet.

The CHAIRMAN congratulated the Meeting on the large amount of interesting information which had been elicited by the discussion. Sir Henry Rawlinson had spoken of the ditch and the fort which were considered by the great Mogul conquerors long ago to be the true defences of Western India. No doubt this view was right, but it was one which Englishmen were only within the last few years apparently beginning to appreciate at its full value. Whatever might be the political results of the present movement of military forces in Afghanistan, it must be very satisfactory to find that at all events a great deal of accurate topographical and scientific knowledge would be obtained.

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*On the Colouring of Maps.* By Professor CAXLEY.

THE theorem that four colours are sufficient for any map, is mentioned somewhere by the late Professor De Morgan, who refers to it as a theorem known to map-makers. To state the theorem in a precise form, let the term "area" be understood to mean a simply or multiply connected\* area: and let two areas, if they touch along a line, be said to be "attached" to each other; but if they touch only at a point or points, let them be said to be "appointed" to each other. For instance, if a circular area be divided by radii into sectors, then each sector is attached to the two contiguous sectors, but it is appointed to the

\* An area is "connected" when every two points of the area can be joined by a continuous line lying wholly within the area; the area within a non-intersecting closed curve, or say an area having a single boundary, is "simply connected"; but if besides the exterior boundary there is one or more than one interior boundary (that is, if there is within the exterior boundary one or more than one *enclave* not belonging to the area), then the area is "multiply connected." The theorem extends to multiply connected areas, but there is no real loss of generality in taking, and we may for convenience take the areas of the theorem to be each of them a simply connected area.

several other sectors. The theorem then is, that if an area be partitioned in any manner into areas, these can be, with four colours only, coloured in such wise that in every case two attached areas have distinct colours; appointed areas may have the same colour. Detached areas may in a map represent parts of the same country, but this relation is not in anywise attended to: the colours of such detached areas will be the same, or different, as the theorem may require.

It is easy to see that four colours are wanted; for instance, we have a circle divided into three sectors, the whole circle forming an *enclave* in another area; then we require three colours for the three sectors, and a fourth colour for the surrounding area: if the circle were divided into four sectors, then for these two colours would be sufficient, and taking a third colour for the surrounding area, three colours only would be wanted; and so in general according as the number of sectors is even or odd, three colours or four colours are wanted. And in any tolerably simple case it can be seen that four colours are sufficient. But I have not succeeded in obtaining a general proof: and it is worth while to explain wherein the difficulty consists. Supposing a system of  $n$  areas coloured according to the theorem with four colours only, if we add an  $(n + 1)$ th area, it by no means follows that we can *without altering the original colouring* colour this with one of the four colours. For instance, if the original colouring be such that the four colours all present themselves in the exterior boundary of the  $n$  areas, and if the new area be an area enclosing the  $n$  areas, then there is not any one of the four colours available for the new area.

The theorem, if it is true at all, is true under more stringent conditions: for instance, if in any case the figure includes four or more areas meeting in a point (such as the sectors of a circle), then if (introducing a new area) we place at the point a small circular area, cut out from and attaching itself to each of the original sectorial areas, it must according to the theorem be possible with four colours only to colour the new figure; and this implies that it must be possible to colour the original figure so that only three colours (or it may be two) are used for the sectorial areas. And in precisely the same way (the theorem is in fact really the same) it must be possible to colour the original figure in such wise that only three colours (or it may be two) present themselves in the exterior boundary of the figure.

But now suppose that the theorem *under these more stringent conditions* is true for  $n$  areas: say that it is possible with four colours only, to colour the  $n$  areas in such wise that not more than three colours present themselves in the external boundary: then it might be easy to prove that the  $n + 1$  areas could be coloured with four colours only: but this would be insufficient for the purpose of a general proof; it would be necessary to show further that the  $n + 1$  areas could be with the four colours only coloured *in accordance with the foregoing boundary condition*; for without this we cannot from the case of the  $n + 1$  areas pass to the

next case of  $n + 2$  areas. And so in general, whatever more stringent conditions we import into the theorem as regards the  $n$  areas, it is necessary to show not only that the  $n + 1$  areas can be coloured with four colours only, but that they can be coloured in accordance with the more stringent conditions. As already mentioned, I have failed to obtain a proof.

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## GEOGRAPHICAL PROFESSORSHIPS AT OXFORD AND CAMBRIDGE.

IN June, 1874, the Council of the Royal Geographical Society addressed, through their President, Sir Bartle Frere, a Memorial to the Vice-Chancellors respectively of Oxford and Cambridge, urging the claims of Geographical science to due recognition in any future distribution of Academical Revenues, and pointing out the expediency of establishing a Professorship of Geography and founding Travelling Scholarships.\* As was stated in the Memorial, no decisive reply was expected until the Royal Commission appointed to investigate Academical revenues should have made its report. This time, it is believed, is now drawing near, and the Council have therefore renewed its application, in the form of a second Memorial, addressed to the Commissioners, and also to the governing bodies of each University. The Memorial, with covering letter, signed by Sir Rutherford Alcock as Vice-President, was despatched on the 28th of February last. It runs as follows:—

While the organisation of Academical studies is under the consideration of the Oxford and Cambridge University Commissions, the Council of the Royal Geographical Society avail themselves of the opportunity thus afforded to urge on the Commissioners, as well as on the Governing Bodies of both Universities, the importance of establishing Geographical Professorships. The claims of Geography to be thus represented appear to the Council to be both weighty and numerous. They are briefly set forth in the following Memorandum.

The Council desire the word Geography to be understood in its most liberal sense, and not as an equivalent to topography. They mean by it, a compendious treatment of all the prominent conditions of a country, such as its climate, configuration, minerals, plants, and animals, as well as its human inhabitants; the latter in respect not only to their race, but also to their present and past history, so far as it is intimately connected with the peculiarities of the land they inhabit.

A Scientific Geographer does not confine himself to descriptions of separate localities, such as may be found in gazetteers, but he groups similar cases together and draws those generalisations from them to which the name of "Aspects of Nature" has been given. He studies the mutual balance and restraint of the various forms of vegetation and of

\* Printed in 'Proceedings R. G. S.,' vol. xviii. p. 451.