Directions: Problems are worth 20 points each. Show your work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answers to each problem (e.g., put a box around them); and clearly separate solutions to each problem from other problems. **Good luck!**

Problem 1:

a. Using the statement letters H, K, A for the component statements, translate the following compound statements into symbolic notation.

i. (2 pts) The knight will win only if the horse is fresh and the armor is strong.

$K \to H \wedge A$ ✓

ii. (2 pts) A fresh horse is a necessary condition for the knight to win.

$K \to H$ ✓

iii. (2 pts) A sufficient condition for the knight to win is that the armor is strong or the horse is fresh.

$H \vee A \to K$ ✓

iv. (4 pts) Write the negation of part i. (and simplify); then state it in words.

$K \wedge (H \wedge A)' = K \wedge (H' \vee A')$ ✓

The knight wins but the horse isn't fresh or the armor isn't strong

good

v. (2 pts) Write the converse of part ii.

$H \to K$ ✓

vi. (2 pts) Write the contrapositive of part iii. (simplify)

$K' \to (H \vee A)' = K' \to (H' \wedge A')$ ✓

b. (6 pts) Prove DeMorgan's law: $(A \vee B)' \leftrightarrow (A' \wedge B')$. ✓

A	B	A'	B'	$(A \vee B)'$	$(A' \wedge B')$	$(A \vee B)' \leftrightarrow (A' \wedge B')$
T	T	F	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

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Problem 2:

a. (10 pts) Consider this wff: $[A \to (B \to C)] \to [B \to (A \to C)]$.

i. Explain why this wff is valid. Using contradiction.

Assume, $A \to (B \to C)$ is true, while $B \to (A \to C)$ is false.

We have: B is true, but $A \to C$ is False. Since $A \to C$ is false, then A is true and C is False. Since B is true and C is false, then $B \to C$ is false. Since A is true and $B \to C$ is false, then $A \to (B \to C)$ is false, which conflicts with the assumption. Therefore, the given wff is valid.

ii. Prove it.

$[A \to (B \to C)] \to [B \to (A \to C)]$

$\Leftrightarrow [A \to (B \to C)] \wedge B \to (A \to C) \Leftrightarrow [A \to (B \to C)] \wedge B \wedge A \to C$

(hyp deduction) (hyp deduction)

1. $A \to (B \to C)$ hyp
2. B hyp
3. A hyp
4. $B \to C$ 1,3 mp
5. C 2,4 mp

Nice work! You proved it twice.

b. (10 pts) If Jose took the jewelry or Mrs. Krasov lied, then a crime was committed. Mr. Krasov was not in town. If a crime was committed, then Mr. Krasov was in town. Therefore Jose did not take the jewelry. Use the statement letters J (Jewelry), L (Lied), C (Crime), T (in Town).

i. Write the argument as a propositional wff.

$[(J \vee L) \to C] \wedge T' \wedge (C \to T) \to J'$ ✓

ii. Prove the argument valid.

1. $(J \vee L) \to C$ hyp
2. T' hyp ✓
3. $C \to T$ hyp
4. C' 2,3 mt ✓
5. $(J \vee L)'$ 1,4 mt
6. $J' \wedge L'$ 5,dm ✓
7. J' 6, simp ✓

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Problem 2:

$[A \to B] \to [A \to C]$?

a. (10 pts) Consider this wff: $[A \to (B \to C)] \to [B \to (A \to C)]$.

i. Explain why this wff is valid.

This is valid because by using exportation you get $[A \wedge B] \to C$ and $[B \wedge A] \to C$ so by Communative Proportey and deduction you get the other.

Okay!

ii. Prove it.

A	B	C	$B \to C$	$A \to (B \to C)$	$A \to C$	$B \to (A \to C)$	$[A \to (B \to C)] \to [B \to (A \to C)]$	
T	T	T	T	T	T	T	T→T	T
T	T	F	F	F	F	F	F→F	T
T	F	T	T	T	T	T	T→T	T
T	F	F	T	T	F	T	T→T	T
F	T	T	T	T	T	T	T→T	T
F	T	F	F	T	T	T	T→T	T
F	F	T	T	T	T	T	T→T	T
F	F	F	T	T	T	T	T→T	T

b. (10 pts) If Jose took the jewelry or Mrs. Krasov lied, then a crime was committed. Mr. Krasov was not in town. If a crime was committed, then Mr. Krasov was in town. Therefore Jose did not take the jewelry. Use the statement letters J (Jewelry), L (Lied), C (Crime), T (in Town).

i. Write the argument as a propositional wff.

$[(J \vee L) \to C] \wedge T' \wedge [C \to T] \to J'$

ii. Prove the argument valid.

1	$(J \vee L) \to C$	HYP
2	T'	HYP
3	$C \to T$	HYP
4	$T' \to C'$	3, Contra
5	C'	2, 4, mp
6	$C' \to (J \vee L)'$	1, Contra
7	$C' \to J' \wedge L'$	6, De Morgan
8	J'	5, 7, mp + Simplification

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roblem 3:

a. (10 pts) Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff in the universe of all people and all times.

$P(x):$ x is a person
$T(x):$ x is a time
$L(x,y):$ x is liked at time y

i. Some people are liked all of the time.

$\exists x\,[P(x) \wedge (\forall y)(T(y) \longrightarrow L(x,y))]$ ✓

ii. All of the people are liked some of the time.

$\forall x\,[P(x) \longrightarrow (\exists y)(T(y) \wedge L(x,y))]$ ✓

iii. All of the people aren't liked all of the time.

$(\forall x)(\forall y)\,[(P(x) \wedge T(y)) \longrightarrow L(x,y)']$ ✓

iv. Negate the predicate wff in part a. (and simplify – you can't just put a "not" on it!:)

$[(\exists x)(P(x) \wedge (\forall y)(T(y) \longrightarrow L(x,y)))]'$
$(\forall x)(P'(x) \vee (\exists y)(T(y) \rightarrow L(x,y))'$ $\rightarrow (T'(y) \vee L(x,y))'$ – implication
$(\forall x)(P'(x) \vee (\exists y)(T(y) \wedge L(x,y)'))$

good

b. (10 pts) For each wff, find an interpretation in which it's true, and one in which it's false, **using a common universe** in each case:

i. $(\forall x)(\forall y)(P(x,y) \rightarrow P(y,x))$ Universe: all people

- True:

P(x,y) = x is a sibling of y
P(y,x) = y is a sibling of x ✓

- False:

P(x,y) = x is a son of y
P(y,x) = y is a son of x ✓

ii. $(\forall x)(P(x) \rightarrow (\exists y)Q(x,y))$ Universe: all Integers

- True:

P(x) = x is an integer
Q(x,y) = x + y = 0 ✓

- False:

P(x) = x is an integer
Q(x,y) = x + y is x

Universe: for all natural no's

✓

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roblem 3:

a. (10 pts) Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff in the universe of all people and all times.

$P(x):$ x is a person
$T(x):$ x is a time
$L(x,y):$ x is liked at time y

i. Some people are liked all of the time.

$(\exists x)[P(x) \wedge (\forall y)[T(y) \rightarrow L(x,y)]]$ ✓

ii. All of the people are liked some of the time.

$(\forall x)[P(x) \rightarrow (\exists y)[T(y) \wedge L(x,y)]]$ ✓

iii. All of the people aren't liked all of the time.

$(\forall x)[P(x) \rightarrow (\exists y)[T(y) \wedge (L(x,y))']]$ ✓

Eric - you're with me! This is how I read it.

iv. Negate the predicate wff in part ~~a~~.i (and simplify – you can't just put a "not" on it!:)

$(\forall x)[P(x) \wedge (\forall y)[T(y) \rightarrow L(x,y)]]'$

$(\forall x)[(P(x))' \vee (\exists y)[(T(y))' \vee L(x,y)]']$

$\boxed{(\forall x)[(P(x))' \vee (\exists y)[T(y) \wedge (L(x,y))']]}$

Well done

b. (10 pts) For each wff, find an interpretation in which it's true, and one in which it's false, **using a common universe** in each case:

i. $(\forall x)(\forall y)(P(x,y) \rightarrow P(y,x))$ Universe: integers

- True: $P(x,y)$: x and y are coprime
  (have no common factors other than one)

- False:
  $P(x,y)$: x/y is a whole number

ii. $(\forall x)(P(x) \rightarrow (\exists y)Q(x,y))$ Universe: integers

- True: $P(x)$: x is even
  $Q(x,y)$: $x/y = 2$

- False: $P(x)$: x is odd
  $Q(x,y)$: $x/y = 2$

Problem 4: do only two of the following (your choice); write "skip" on the third.

a. Prove that this wff is a valid argument: $(\forall x)P(x) \wedge (\exists x)Q(x) \to (\exists x)[P(x) \wedge Q(x)]$

1. $(\forall x) P(x)$ hyp
2. $(\exists x) Q(x)$ hyp
3. $Q(a)$ 2; ei
4. $P(a)$ 1; ui
5. $P(a) \wedge Q(a)$ 3&4; conj
6. $(\exists x)[P(x) \wedge Q(x)]$ 5; eg

b. Prove, or give an interpretation in which this is false: $(\forall x)[A(x) \to B(x)] \to [(\exists x)A(x) \to (\exists x)B(x)]$

By deduction method:-

$(\forall x)[A(x) \to B(x)] \wedge (\exists x) A(x) \to (\exists x) B(x)$

1. $(\forall x)[A(x) \to B(x)]$ hyp
2. $(\exists x) A(x)$ hyp
3. $A(a)$ 2; ei
4. $A(a) \to B(a)$ 1; ui
5. $B(a)$ 3&4; mp
6. $(\exists x) B(x)$ 5; eg

b. Prove, or give an interpretation in which this is false: $(\forall x)[A(x) \to B(x)] \to [(\exists x)A(x) \to (\exists x)B(x)]$

1. $(\forall x)[A(x) \to B(x)]$ hyp
2. $(\exists x)A(x)$ hyp, deduction
3. $A(a)$ 2, ei
4. $A(a) \to B(a)$ 1, ui
5. $B(a)$ 3,4, mp
6. $(\exists x)B(x)$ 5, eg

guud

c. Using predicate logic prove the following argument is valid: Everyone with red hair has freckles. Someone has red hair and big feet. Everybody who doesn't have green eyes doesn't have big feet. Therefore someone has green eyes and freckles. $RH(x), Fr(x), BF(x), GE(x)$

$(\forall x)[RH(x) \to Fr(x)] \wedge (\exists x)[RH(x) \wedge BF(x)] \wedge (\forall x)[GE(x)' \to BF(x)']$
$\to (\exists x)[GE(x) \wedge Fr(x)]$

1. $(\forall x)[RH(x) \to Fr(x)]$ hyp
2. $(\exists x)[RH(x) \wedge BF(x)]$ hyp
3. $(\forall x)[GE(x)' \to BF(x)']$ hyp
4. $RH(a) \wedge BF(a)$ 2, ei
5. $RH(a) \to Fr(a)$ 1, ui
6. $GE(a)' \to BF(a)'$ 3, ui
7. $RH(a)$ 4, sim
8. $BF(a)$ 4, sim
9. $Fr(a)$ 5,7, mp
10. $GE(a)$ 6,8, mt
11. $GE(a) \wedge Fr(a)$ 9,10, con
12. $(\exists x)[GE(x) \wedge Fr(x)]$ 11, eg

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Problem 4: do only two of the following (your choice); write "skip" on the third.

a. Prove that this wff is a valid argument: $(\forall x)P(x) \wedge (\exists x)Q(x) \to (\exists x)[P(x) \wedge Q(x)]$

1. $(\forall x)\, P(x)$	hyp
2. $(\exists x)\, Q(x)$	hyp
3. $Q(a)$	2, ei
4. $P(a)$	1, ui
5. $P(a) \wedge Q(a)$	3,4 conj
$\boxed{6.\ (\exists x)[P(x) \wedge Q(x)]}$	5, eg

b. Prove, or give an interpretation in which this is false: $(\forall x)[A(x) \to B(x)] \to [(\exists x)A(x) \to (\exists x)B(x)]$

Skip

c. Using predicate logic prove the following argument is valid: Everyone with red hair has freckles. Someone has red hair and big feet. Everybody who doesn't have green eyes doesn't have big feet. Therefore someone has green eyes and freckles. $RH(x), Fr(x), BF(x), GE(x)$

$(\forall x)(RH(x) \to Fr(x)) \wedge (\exists x)(RH(x) \wedge BF(x)) \wedge (\forall x)(GE(x)' \to BF(x)') \to (\exists x)(GE(x) \wedge Fr(x)$

1. $(\forall x)\, RH(x) \to Fr(x)$	hyp
2. $(\exists x)\, RH(x) \wedge BF(x)$	hyp
3. $(\forall x)\, GE(x)' \to BF(x)'$	hyp
4. $RH(a) \wedge BF(a)$	2, ei
5. $RH(a) \to Fr(a)$	1, ui
6. $GE(a)' \to BF(a)'$	3, ui
7. $BF(a) \to GE(a)$	6, contraposition
8. $RH(a)$	4, simp
9. $BF(a)$	4, simp
10. $GE(a)$	9,6, mt
11. $RH(a)' \vee Fr(a)$	5, imp
12. $Fr(a)$	8,11 ds
13. $GE(a) \wedge Fr(a)$	10,12 conj
$\boxed{14.\ (\exists x)[GE(x) \wedge Fr(x)]}$	13, eg

good work!

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oblem 5: : If the sum of two integers x and y is not divisible by integer n, then one or the other of the two integers is not divisible by n.

a. (10 pts) Prove by contradiction. $[(x+y)|n]' \to [(x|n)' \vee (y|n)']$

Let x+y not be divisible by n.
Assume x is divisible by n and y is divisible by n.
Then, by definition of divisible

$x = jn$
$y = kn,$

for some $j, k \in \mathbb{Z}$. By substitution,

$x+y = jn + kn$
$= (j+k)n,$

where j+k is an integer by the closure of integers under addition.
Thus, by the definition of divisible, x+y is divisible by n,
which is a contradiction.
Therefore, either x is not divisible by n or y is not divisible by n.

b. (10 pts) Prove by contraposition. $[(x|n) \wedge (y|n)] \to (x+y|n)$

Let x be divisible by n and y be divisible by n.
By the definition of divisible,

$x = jn$
$y = kn,$

where j and k are integers.
By substitution,

$x+y = jn + kn$
$= (j+k)n,$

where j+k is an integer by the closure of integers under addition.
Thus, x+y is divisible by n, as desired.

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Problem Extra Credit (4 pts): : Lewis Carroll gave the following syllogism:

"Some new Cakes are unwholesome;
no nice Cakes are unwholesome;
therefore some new Cakes are not-nice."

Use his diagrams and methods of the logic game to illustrate how one arrives at the conclusion from the premises.