Chapter 2, Section 2.2: Preview Activity

Preview Activity 2.2.1. Consider the function $g(x) = 2^x$, which is graphed in Figure 2.2.1 .

a. At each of $x = -2, -1, 0, 1, 2$, use a straightedge to sketch an accurate tangent line to $y = g(x)$.

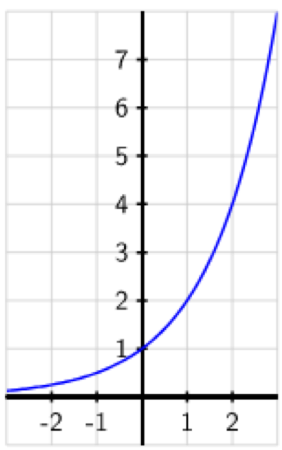
b. Use the provided grid to estimate the slope of the tangent line you drew at each point in (a).

c. Use the limit definition of the derivative to estimate $g'(0)$ by using small values of h, and compare the result to your visual estimate for the slope of the tangent line to $y = g(x)$ at $x = 0$ in (b).

d. Based on your work in (a), (b), and (c), sketch an accurate graph of $y = g'(x)$ on the axes adjacent to the graph of $y = g(x)$.

e. Write at least one sentence that explains why it is reasonable to think that $g'(x) = cg(x)$, where c is a constant. In addition, calculate $\ln(2)$, and then discuss how this value, combined with your work above, reasonably suggests that $g'(x) = 2^x \ln(2)$.

In[1]:=



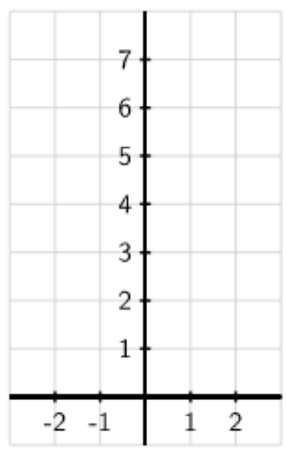
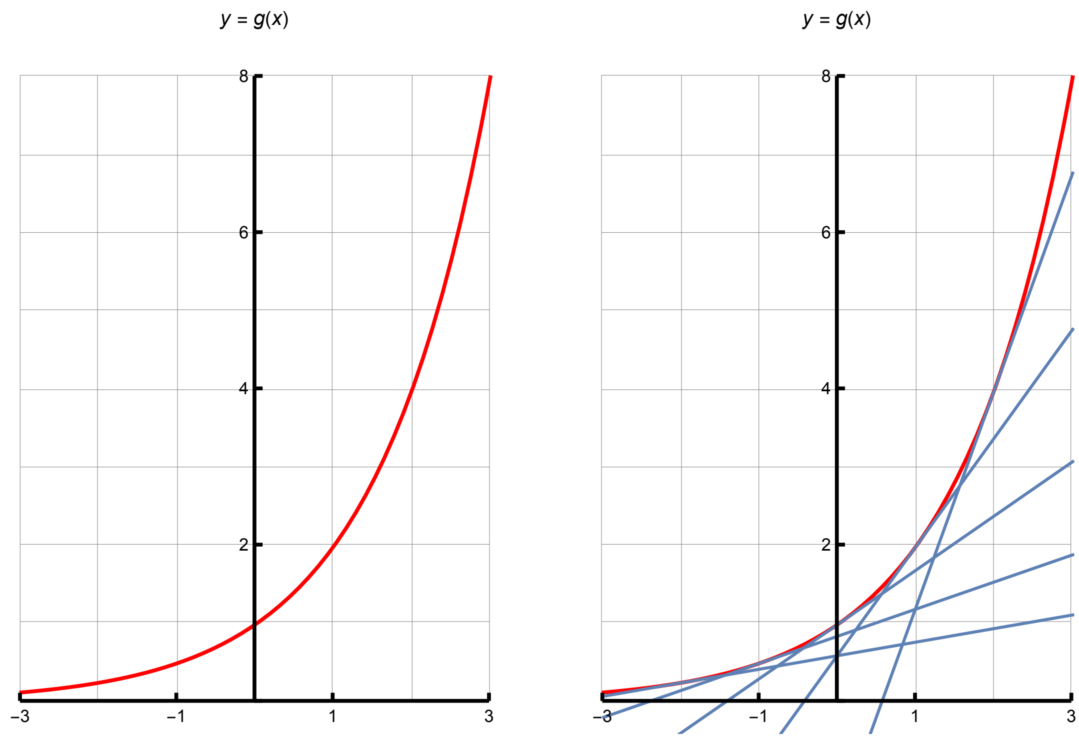


Figure 2.2.1. At left, the graph of $y = g(x) = 2^x$. At right, axes for plotting $y = g'(x)$.

a. At each of x=−2,−1,0,1,2, use a straightedge to sketch an accurate tangent line to y=g(x).

Out[99]=



b. Use the provided grid to estimate the slope of the tangent line you drew at each point in (a).

In[56]:=
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TableForm[Table[{g'[k], N[g'[k]]}, {k, -2, 2}]]
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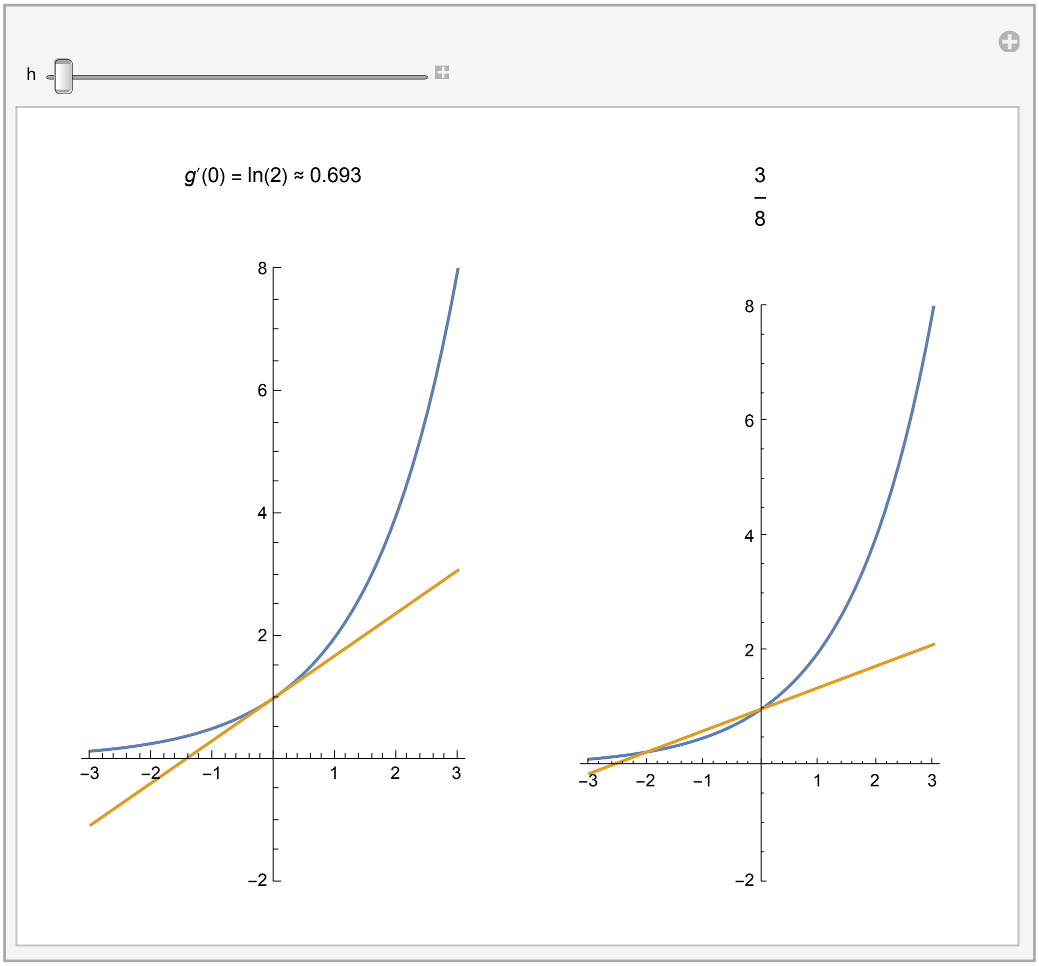
Out[56]//TableForm=

$\frac{\text{Log}[2]}{4}$	0.173287
$\frac{\text{Log}[2]}{2}$	0.346574
Log[2]	0.693147
2 Log[2]	1.38629
4 Log[2]	2.77259

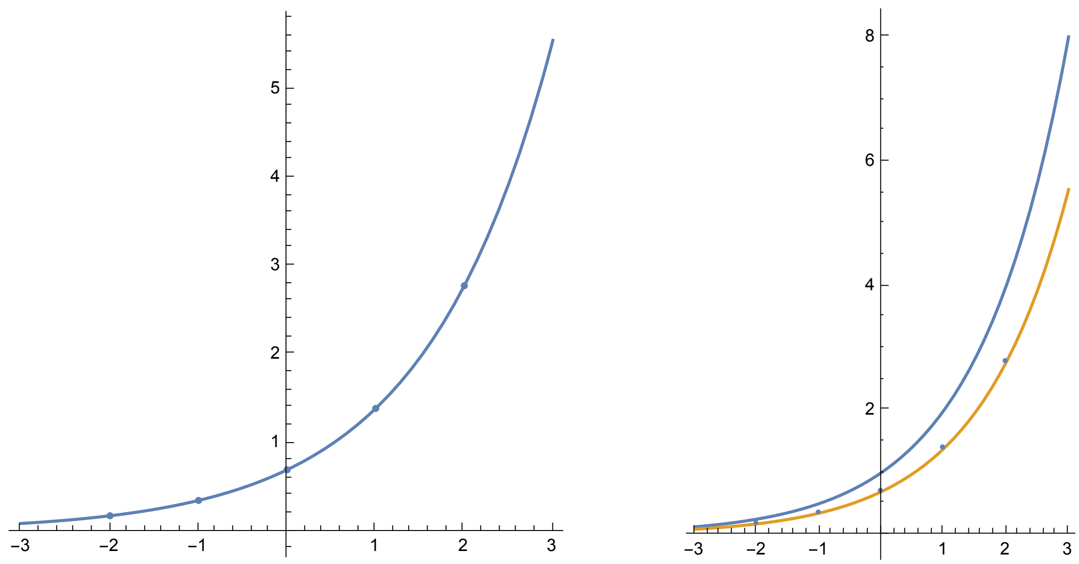
c. Use the limit definition of the derivative to estimate g'(0)
by using small values of h, and compare the result to your visual estimate for the slope of the tangent line to y=g(x) at x=0 in (b).

Out[73]=



d. Based on your work in (a), (b), and (c), sketch an accurate graph of y=g’(x) on the axes adjacent to the graph of y=g(x).

Out[91]=



e. Write at least one sentence that explains why it is reasonable to think that g'(x)=cg(x), where c is a constant. In addition, calculate ln(2), and then discuss how this value, combined with your work above, reasonably suggests that g'(x)=2^x ln(2).

It seems that there’s a common ratio between the values of g and the values of g’, which is about .7. That is approximately the value of ln(2).