Quiz 3, MAT128 -- Spring, 2023

Name:

Exercise 1 (2 pts):

Write the formula for a limit definition of the derivative of the function f at the point x=a below. There are several to choose from, but you know which one I like best!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Good

Exercise 2 (4 pts):

Use the limit definition to compute the derivative of f(x)=x^2 at the point x=3.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x + \cancel{h}$$

$$= 2x$$

$\therefore$ at $x = 3$

$$f'(3) = 2(3)$$

$$= 6$$

Exercise 3 (4 pts):

Activity 1.3.2. Consider the function f whose formula is $f(x) = 3 - 2x$.

$3-2(5+h)$
$3-10+2h$
$-7+2h$

a. What familiar type of function is f? What can you say about the slope of f at every value of x?

b. Compute the average rate of change of f on the intervals $[1, 4]$, $[3, 7]$, and $[5, 5+h]$; simplify each result as much as possible. What do you notice about these quantities?

c. Use the limit definition of the derivative to compute the exact instantaneous rate of change of f with respect to x at the value $a = 1$. That is, compute $f'(1)$ using the limit definition. Show your work. Is your result surprising?

d. Without doing any additional computations, what are the values of $f'(2)$, $f'(\pi)$, and $f'(-\sqrt{2})$? Why?

a. This function's graph is a line. The slope is constantly -2.

It's an affine function. sometimes called linear.

b. $[1,4] = (1,1) \& (4,-5) = \frac{-5-1}{4-1} = -2.$

$[3,7] = (3,-3) \& (7,-11) = \frac{-11+3}{7-3} = -2.$

$[5,5+h] = (5,-7) \& (5+h, -7+2h) = \frac{-7+2h+7}{5+h-5} = -2.$

All of these quantities equal -2, the slope of the overall line.

c. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{(3-2(x+h)) - (3-2x)}{h}$$
$$= \lim_{h\to 0} \frac{(\cancel{3}-\cancel{2x}-2h)-\cancel{3}+\cancel{2x}}{h}$$
$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}}$$
$$= -2$$

This result is not surprising, as the slope never changes.

d. The values of all x-values, like $f'(2)$, $f'(\pi)$, and $f'(-\sqrt{2})$ are -2, as the slope never changes in a line.

Well Done