1. (2 pts) Write the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

2. (6 pts) Use the limit definition to find the derivative of $f(x) = 3x^2 - x$.

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 - (x+h) - [3x^2 - x]}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - x - h - [3x^2-x]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-x} - h \cancel{-3x^2} \cancel{+x}}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 - h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6x+3h-1)}{\cancel{h}}$$

$$= \lim_{h\to 0} \left(6x + 3h - 1\right)$$

use parentheses

$$= 6x - 1$$

2. (6 pts) Use the limit definition to find the derivative of $f(x) = 3x^2 - x$.

$$F'(x) = \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - (x+h) - 3x^2 + x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-x} - h \cancel{-3x^2} \cancel{+x}}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 - h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6x + \cancel{3h} - 1)}{\cancel{h}}$$

not yet

$$= 6x - 1$$

$$= \lim_{h\to 0} (6x + 3h - 1)$$

Now let $h \to 0$ ....

check

$(3\cdot 2)x^{(2-1)} - 1$

$6x - 1$

good!

3. (2 pts) Find the second derivative $f''(x)$ by any means. Describe what it tells you about the graph of the function $f(x)$.

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \lim_{h \to 0} \frac{6(x+h) - 1 - [6x-1]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{6x} + 6h \cancel{-1} - \cancel{6x} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{6\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 6$$

$$= 6$$

This tells me that the graph of $f(x)$ will be concave up, based on the sign of 6, which is positive. This also tells me $f(x)$ is a parabola, which leads to $f'(x)$ being a linear increase and $f''(x)$ being constant.

Nice work