MAT128 Quiz 09, Spring 2023

10/10

Name:

Compute the following derivatives, using the standard differentiation rules and the chain rule. If there are places where the derivative is not defined, mention them.

1. $a(x) = \sqrt{e^x}$

$(f(g(x))' = f'(g(x)) \cdot g'(x)$

$f(x) = \sqrt{x} = x^{1/2}$

$g(x) = e^x$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$g'(x) = e^x$

$\frac{1}{2\sqrt{e^x}} \cdot e^x = a'(x)$ ✓

$a'(x)$ is not defined when x equals 0.

yes it is!

2. $b(x) = \sin(\cos(x))$

$f(x) = \sin(x)$

$g(x) = \cos(x)$

$f'(x) = \cos(x)$

$g'(x) = -\sin(x)$

$b'(x) = \cos(\cos(x)) \cdot -\sin(x)$ ✓

Compute the following derivatives, using the standard differentiation rules and the chain rule. If there are places where the derivative is not defined, mention them.

1. $a(x) = \sqrt{e^x}$

$f(x) = x^{\frac{1}{2}}$ $\quad g(x) = e^x$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $\quad g'(x) = e^x$

$\frac{1}{2}(e^x)^{-\frac{1}{2}} \cdot e^x$ ✓

2. $b(x) = \sin(\cos(x))$

$f(x) = \sin(x)$ $\quad g(x) = \cos(x)$

$f'(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

$-\sin(x) \cdot \cos(\cos(x))$ ✓

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

3. $c(x) = e^{(x^3)}$

$f(x) = e^x$

$g(x) = x^3$

$f'(x) = e^x$

$g'(x) = 3x^2$ → Pwr Rule

$c'(x) = e^{x^3} \cdot 3x^2$ ✓

4. $d(x) = \left(\frac{2x-3}{x}\right)^3$

$a(x) = 2x-3$, $b(x) = x$

$\frac{a'(x)b(x) - a(x)b'(x)}{b^2(x)}$

$f(x) = x^3$

$g(x) = \frac{2x-3}{x}$

$f'(x) = 3x^2$

$g'(x) = \frac{2(x) - (2x-3)(1)}{x^2}$

$= \frac{3}{x^2}$

$d'(x) = 3\left(\frac{2x-3}{x}\right)^2 \cdot \frac{3}{x^2}$ ✓

$d'(x)$ is not defined when $x = 0$. ✓

3. $c(x) = e^{(x^3)}$

$f(x) = e^x \quad g(x) = x^3$

$f'(x) = e^x \quad g'(x) = 3x^2$

$3x^2e^{x^3}$ ✓

4. $d(x) = \left(\frac{2x-3}{x}\right)^3$

$f(x) = x^3$

$f'(x) = 3x^2$

$g(x) = \frac{2x-3}{x}$

$h(x) = 2x-3 \quad j(x) = x$

$h'(x) = 2 \quad j'(x) = 1$

$g'(x) = \frac{2 \cdot x - (2x-3) \cdot 1}{x^2}$

$g'(x) = \frac{2x-2x+3}{x^2}$

$g'(x) = \frac{3}{x^2}$

$3\left(\frac{2x-3}{x}\right)^2 \cdot \frac{3}{x^2}$

$3\left(\frac{2x-3}{x}\right)^2 \cdot \frac{1}{x^2} \cdot 3$

$3\left(\frac{1}{x^2}\left(\frac{2x-3}{x}\right)^2\right)$ ✓

The derivative is not defined at $x = 0$