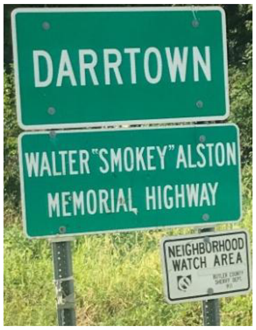
UNIT 2 LIMITS

LIMITS (The Road to Darrtown)



Darrtown, Ohio is located 8 miles from Oxford, Ohio. A cricket sets out from Oxford and walks half way to Darrtown. She stops for the night and the next morning decides to walk half the remaining distance before stopping for the night. She repeats this over and over, walking half the remaining distance each day. The cricket can see the city limits sign for Darrtown. She gets closer and closer. **She knows where the limit is even though she never reaches it.** Math limits are similar. You can figure out what they are without actually reaching them.

To illustrate a typical limit problem let's look at the ratio R(h)

$$R(h) = \frac{(3+h)^2 - 9}{h}$$

We want to figure out the limit of R(h) as h approaches zero. We can't do this by setting h equal to zero because that gives zero divided by zero

$$R(0) = \frac{(3+0)^2 - 9}{0} = \frac{9-9}{0} = \frac{0}{0}$$

We can prove that the limit value is 6 by expanding the binomial in the numerator.

$$(3 + h)^2 = 3^2 + 6h + h^2 = 9 + 6h + h^2$$

This converts R(h) into

$$R(h) = \frac{(3+h)^2 - 9}{h} = \frac{\cancel{9} + 6h + h^2 - \cancel{9}}{h}$$

The nines cancel and **we factor h, which is then canceled by the h in the denominator**

$$R(h) = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6 + h$$

Canceling the factor h gets rid of the divide by zero issue.

The equation R(h) = 6 + h shows us that R(h) approaches 6 as h approaches zero. The symbolism used for the limit process is

$$\lim_{h\to 0} R(h) = \lim_{h\to 0} (6 + h) = 6$$

Math Speak: The limit of 6 + h as h **approaches** zero equals 6.

Like the cricket who can see the city limits sign without reaching it, we can see the limit of 6 + h without setting h = 0.

Example. Evaluate $\lim_{h\to 0} \frac{6}{(6+h)}$

There is no divide by zero issue. The denominator approaches 6 as h approaches zero and the ratio approaches $\frac{6}{6} = 1$.

Exercise 23. Evaluate the limits

A) $\lim_{h\to 0} \frac{4}{(1+2h)}$ B) $\lim_{h\to 0} \frac{(8x+h^2)}{(4+3h)}$

C) $\lim_{h\to 0} \frac{(4+h)^2 - 16}{h(4+3h)}$ D) $\lim_{N\to \infty} (1 + \frac{1}{N})$