Directions: Problems are worth 20 points each; you will do five. You **must** skip **two** of the problems 2-7 (write "SKIP" **clearly** on them). Show your work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). **Indicate clearly your answers to each problem** (e.g., put a box around your answers). **Good luck!**

Problem 1 (you may not skip this one!):

a. (10 pts) Use the limit definition of the derivative to calculate the derivative of the function $f(x) = \dfrac{2}{x^2}$ at the point $x = 2$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h} \cdot \frac{(x+h)^2x^2}{(x+h)^2x^2}$$

$2x^{-2} = -4x^{-3}$

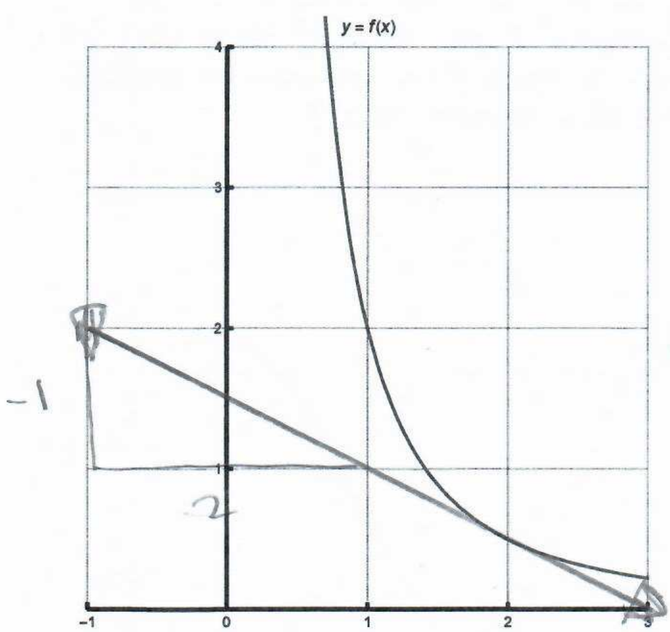
$$= \lim_{h\to 0} \frac{2x^2 - 2(x+h)^2}{h\,x^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{2x^2 - (2x^2+4xh+2h^2)}{hx^2(x^2+2xh+h^2)} = \lim_{h\to 0} \frac{\cancel{2x^2} - \cancel{2x^2} - 4xh - 2h^2}{h(x^4+2x^3h+h^2x^2)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-4x-2h)}{\cancel{h}(x^4+3x^3h+hx^2)} = \frac{-4x}{x^4} = \boxed{\frac{-4}{x^3} = f'(x)}$$

$$f'(2) = \frac{-4}{(2)^3} = \frac{-4}{8} = \boxed{-\frac{1}{2} = f'(2)}$$

b. Carefully estimate the derivative at $x = 2$ using the graph: how well did you do in part a?



$$\text{Slope} = \frac{2-1}{-1-1} = -\frac{1}{2}$$

$\therefore$ the derivative or instantaneous velocity at $x = 2$ is $-\frac{1}{2}$

20

Problem 1 (you may not skip this one!):

a. (10 pts) Use the limit definition of the derivative to calculate the derivative of the function $f(x) = \frac{2}{x^2}$ at the point $x = 2$.

limit definition,

Here, $f(x) = 1/2$ $\quad f(x+h) : \frac{2}{4+4h+h^2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

good

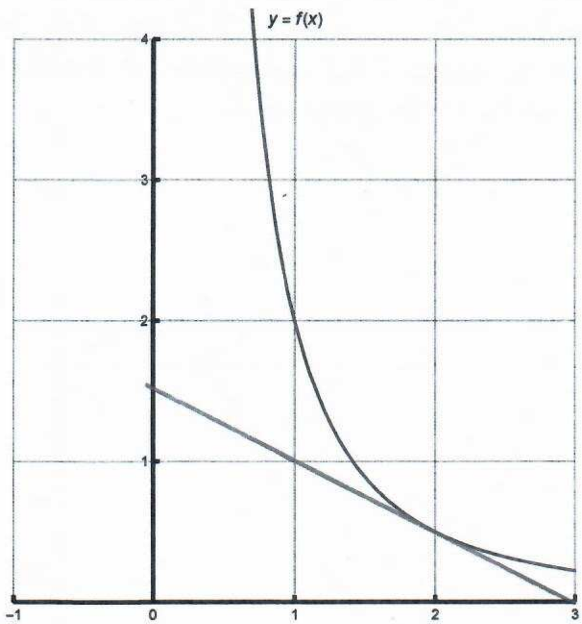
$$f'(x) = \lim_{h \to 0} \frac{\frac{2}{4+4h+h^2} - \frac{1}{2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{\cancel{4} - \cancel{4} - 4h - h^2}{8+8h+2h^2}}{h} \qquad f'(x) = \lim_{h \to 0} \frac{\cancel{h}(-h-4)}{(8+8h+2h^2)\cancel{h}}$$

$$f'(x) = \frac{-4}{8} = -\frac{1}{2}$$

$\therefore$ derivative of the function at $x = 2$ is $-1/2$.

b. Carefully estimate the derivative at $x = 2$ using the graph: how well did you do in part a?



$(x_1, y_1) = (3, 0)$

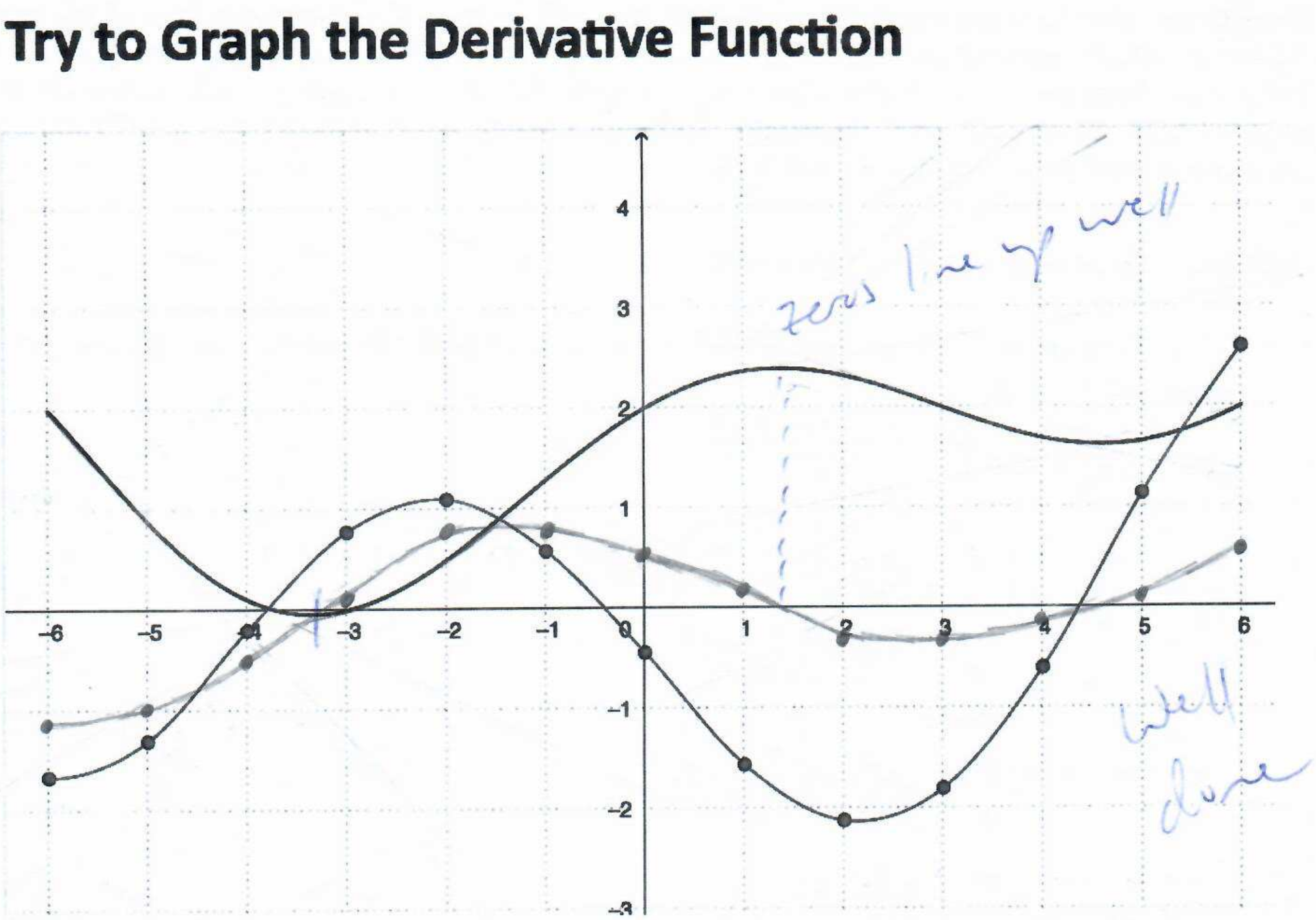
$(x_2, y_2) = (0, 1.5)$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.5 - 0}{0 - 3} = \frac{1.5}{-3}$$

$$= -1/2$$

$\therefore$ The derivative of the function at $x = 2$ is equal to the slope of the tangent line at $x = 2$

Problem 2: You might recall this activity from your text. The smooth curve (in black) is the graph of a function $f(x)$, and you are to correct the (rather poor) attempt at its derivative (with the dots along it):



a. (12 pts) By using your ruler, and **carefully** estimating tangent line slopes, give a much improved approximation to the derivative function (especially at the integer values, where the dots appear on the "pretender" to the derivative function). Finish by lightly connecting your estimates with a smooth curve, representing the derivative function.

b. (8 pts) Discuss the sign (and zeros) of the second derivative of the function f above (and the implications for its graph). In particular, give intervals on which $f''(x)$ is positive or negative, and the places where $f''(x) = 0$, and explain what each implies for the graph of f.

i. $f''(x) > 0$ Intervals [-6,-1] and [3, 6], meaning these intervals are concave up on the graph of f.

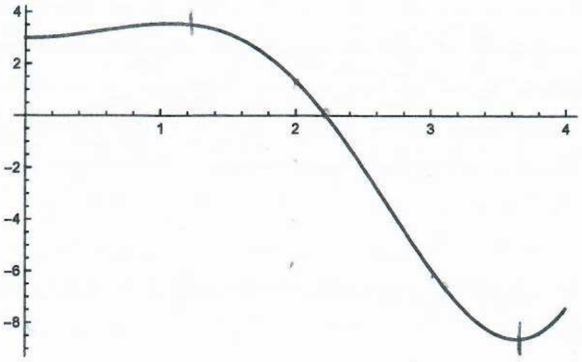
ii. $f''(x) = 0$ Points -1 and 3 would be zero, meaning these points are where concavity changes on the graph of f, or its inflection points.

derivative should have max at x = -1 & min at x = 3 - close!

iii. $f''(x) < 0$ Interval [-1, 3], meaning this interval is concave down on the graph of f.

20

Problem 3: The graph of the function f is given below, along with four potential derivatives. For three of the graphs A-D, explain how you can reject the graph (one is them is derivative).



a. although the derivative graph appears to be correct, the units of the y-axis are far to great for this to be it.

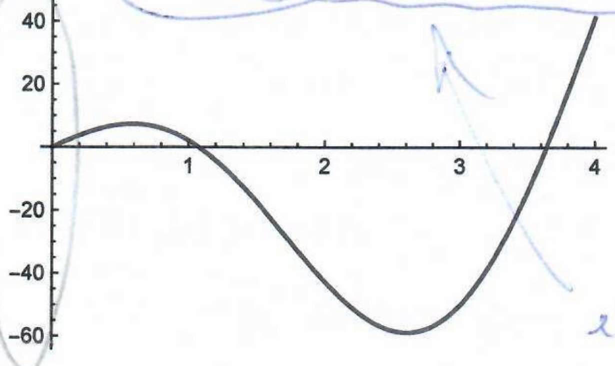
b. this is correct because it has the proper zeros, the graph matches the instantaneous velocity at each point, and the y-axis units are correct.

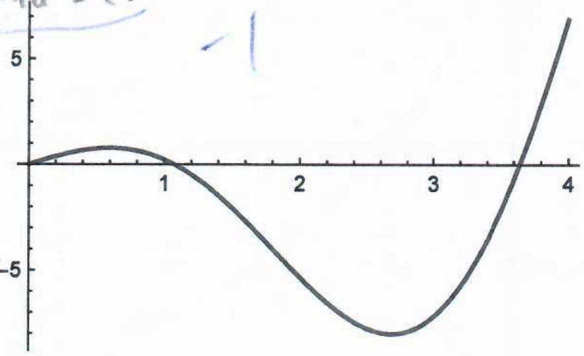
c. like a. the units of the y-axis are far to large for this to be the derivative graph also, at $x=4$, the derivative makes it appear that f is beginning to curve of but that is not shown in f(x).

d. ↳ y-axis units too large and the graphs values are opposite of what they should be.

a b

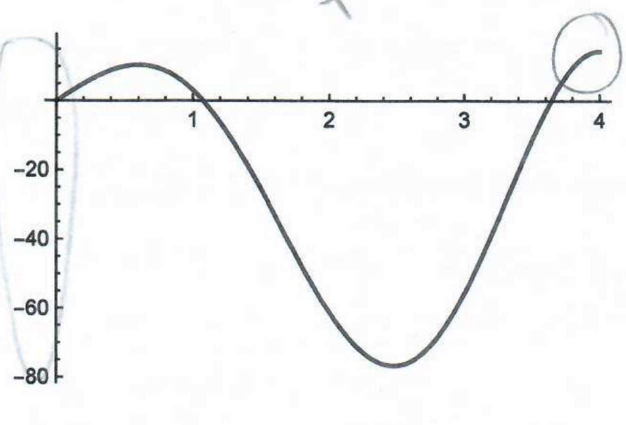
-1

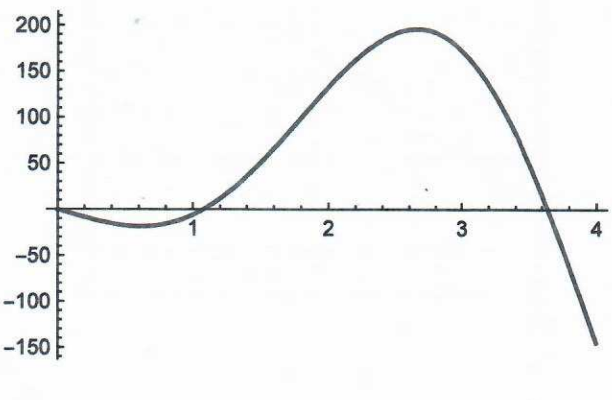




explain

c d





19