Examples of the Chain Rule
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Calculus I

These examples are from Stewart's Calculus:

#9, p. 154
#16
#29
#42
#77

```
In[1]:= Clear["Global`*"]
```

#9

```
In[2]:= F[x_] := Sqrt[1 - 2 x]
     F'[x]
     (* Now let's identify the composition: *)
     f[x_] := Sqrt[x]
     g[x_] := 1 - 2 x
     f'[g[x]] × g'[x] (* same thing! Thank goodness.... *)
```

$$Out[3]= -\frac{1}{\sqrt{1 - 2 x}}$$

$$Out[6]= -\frac{1}{\sqrt{1 - 2 x}}$$

#16

```
In[7]:= Clear[n]
     F[x_] := 6 Cot[n x]
     F'[x]
     (* Now let's identify the composition: *)
     f[x_] := 6 Cot[x]
     g[x_] := n x
     f'[g[x]] × g'[x] (* same thing! Thank goodness.... *)
```

$$Out[9]= -6 n Csc[n x]^2$$

$$Out[12]= -6 n Csc[n x]^2$$

#29: this is an interesting example of more than two functions in a chain....

In[13]:= `F[x_] := Sin[Sqrt[1 + x^2]]`
`F'[x] (* = f(g(h(x))) *)`
`(* Now let's identify the composition: three functions this time! *)`
`f[x_] := Sin[x]`
`g[x_] := Sqrt[x]`
`h[x_] := 1 + x^2`
`f'[g[h[x]]] × g'[h[x]] × h'[x] (* same thing! Thank goodness.... *)`

Out[14]= $\dfrac{x \, \mathrm{Cos}\left[\sqrt{1 + x^2}\right]}{\sqrt{1 + x^2}}$

Out[18]= $\dfrac{x \, \mathrm{Cos}\left[\sqrt{1 + x^2}\right]}{\sqrt{1 + x^2}}$

#77

In[19]:= `B[t_] := 4.0 + 0.35 Sin[2 Pi t / 5.4]`
`B'[t]`
`B'[1]`
`(* Now let's identify the composition: *)`
`f[x_] := 4.0 + 0.35 Sin[x]`
`g[t_] := 2 Pi t / 5.4`
`f'[g[t]] × g'[t] (* same thing! Thank goodnes.... *)`
`f'[g[1]] × g'[1]`

Out[20]= `0.407243 Cos[1.16355 t]`

Out[21]= `0.161301`

Out[24]= `0.407243 Cos[1.16355 t]`

Out[25]= `0.161301`