Section 1.4 - The Derivative as a function

There are actually several different ways that one typically estimates a derivative at a point using a table of values. The one that we have used so far is called the forward difference, and gives the average rate of change between the two points (a+h,f(a+h)) and (a,f(a)) as

$$AV_{[a,a+h]} = \frac{f(a+h) - f(a)}{h}.$$

We can consider it an approximation to f'(a). Alternatively however, we could have considered the

backward difference: $\frac{f(a)-f(a-h)}{h}$

or the

centered difference: $\frac{f(a+h)-f(a-h)}{2h}$

Each of these is also an **approximation** to the derivative at a, f'(a). We examine how well each of these works below.

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1. Consider the function given by $f(x) = x^3$:

a. Fill in the following table with the appropriate derivative approximations at the particular value of x, wherever possible (write "NA" if a cell cannot be calculated; fill in the last row after doing part c.):

Out[176]//TableForm=

x	-2.	-1.5	-1.	-0.5	0.	0.5	1.
y=f(x)	-8.	-3.375	-1.	-0.125	0.	0.125	1.
forward	9.25	4.75	1.75	0.25	0.25	1.75	4.75
backward	15.25	9.25	4.75	1.75	0.25	0.25	1.75
centered	12.25	7.	3.25	1.	0.25	1.	3.25
f'(x)	12.	6.75	3.	0.75	0.	0.75	3.

b. We can also use a graph of the function to estimate the derivative function: just draw tangent lines, and estimate slopes of those tangent lines. Fill in this table by estimating slopes of tangent lines at the particular values of x, using the graph of f. Fill in the last row after doing part c.

Out[177]//TableForm=

x	-2.	-1.5	-1.	-0.5	0.	0.5	1.
y=f(x)	-8.	-3.375	-1.	-0.125	0.	0.125	1.
slope	12.	6.75	3.	0.75	0.	0.75	3.
f'(x)	12.	6.75	3.	0.75	0.	0.75	3.

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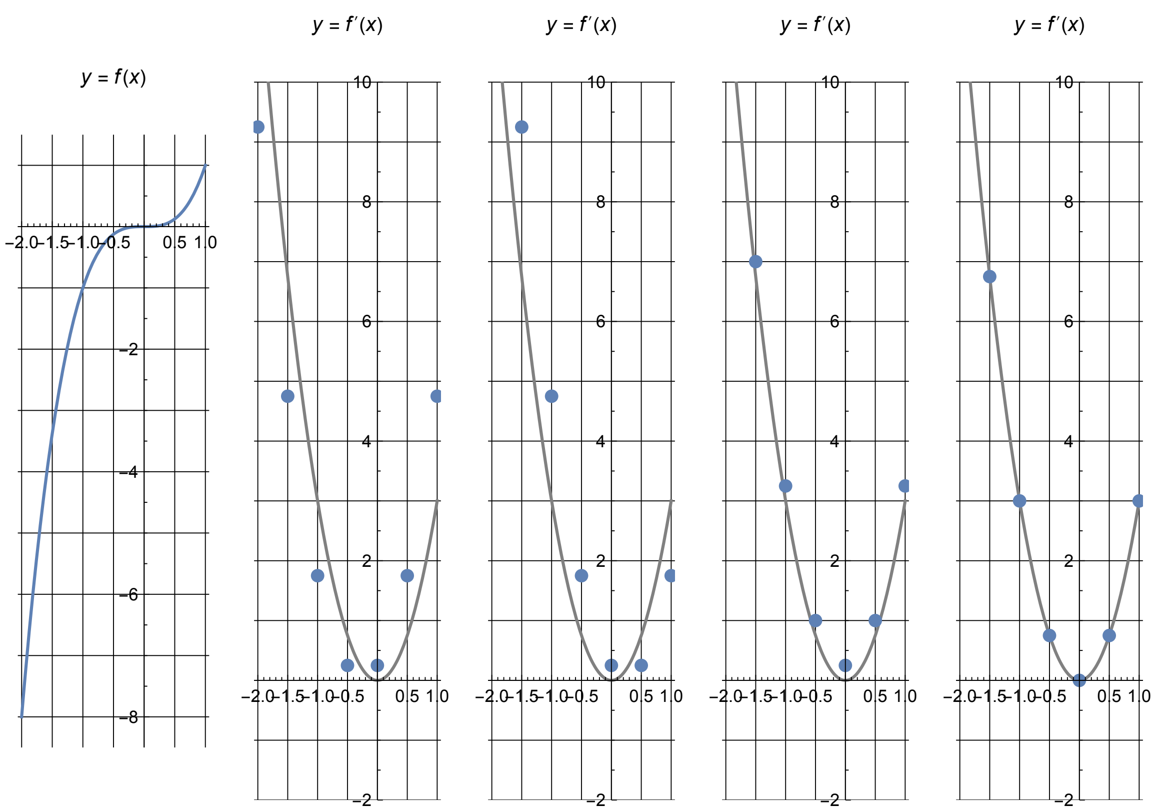

c. Finally we can use algebra to find the derivative function of f(x). Let's do so! We'll make use of Pascal's triangle. At right, draw in the derivative function, and plot the estimates from parts a and b. How does each method do?

2. Consider the function given by $f(x) = x^2 - 3x - 4$:

a. Fill in the following table with the appropriate derivative approximations at the particular value of x, wherever possible (write "NA" if a cell cannot be calculated; fill in the last row after doing part c.):

Out[185]//TableForm=

x	0	1	2	3	4	5
y=f(x)	-4	-6	-6	-4	0	6
forward	-2	0	2	4	6	8
backward	-4	-2	0	2	4	6
centered	-3	-1	1	3	5	7
f'(x)	-3	-1	1	3	5	7

b. We can also use a graph of the function to estimate the derivative function: just draw tangent lines, and estimate slopes of those tangent lines. Fill in this table by estimating slopes of tangent lines at the particular values of x, using the graph of f. Fill in the last row after doing part c.

Out[186]//TableForm=

x	0	1	2	3	4	5
y=f(x)	-4	-6	-6	-4	0	6
slope	-3	-1	1	3	5	7
f'(x)	-3	-1	1	3	5	7

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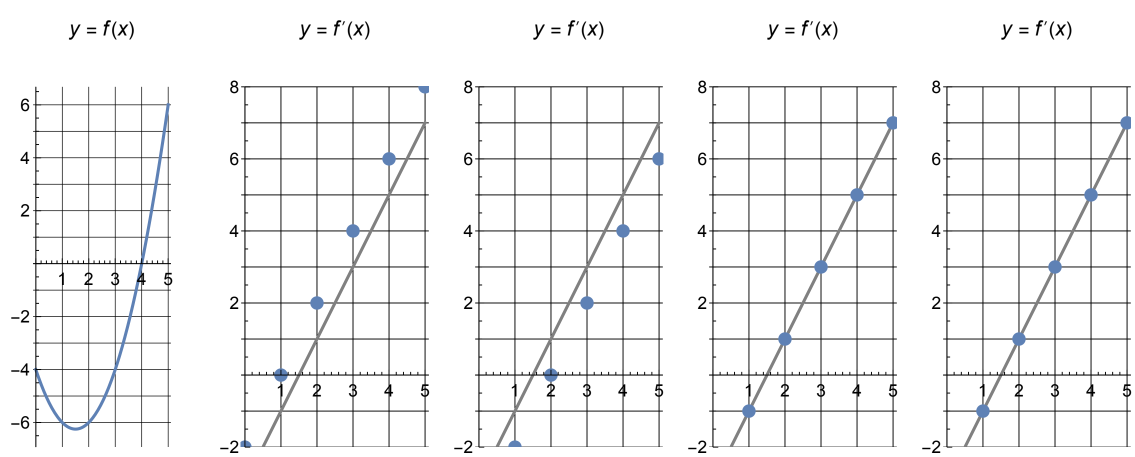
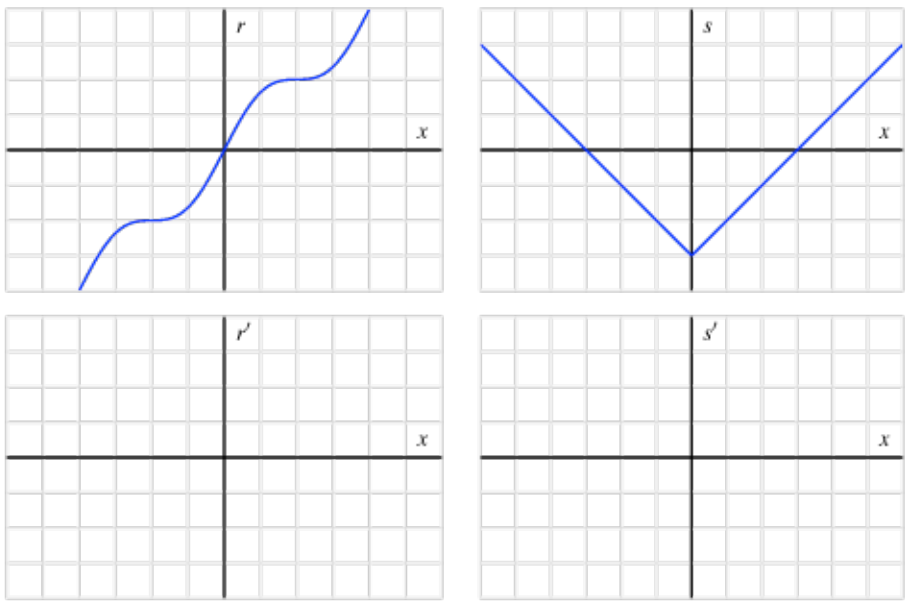


c. Use algebra to find the derivative function of f(x). At right, draw in the derivative function, and plot the estimates from parts a and b. How does each method do?

3. Can you guess a good approximation to the derivative function of these function?

Assume the scale of the grid is 1x1



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