3.3: Global Optimization Worksheet

1. A weight is hung on a series of springs and a strobe light is flashed and a camera records the height above the floor in feet for the weight every half second while the weight bounces. That information is given in the table below.

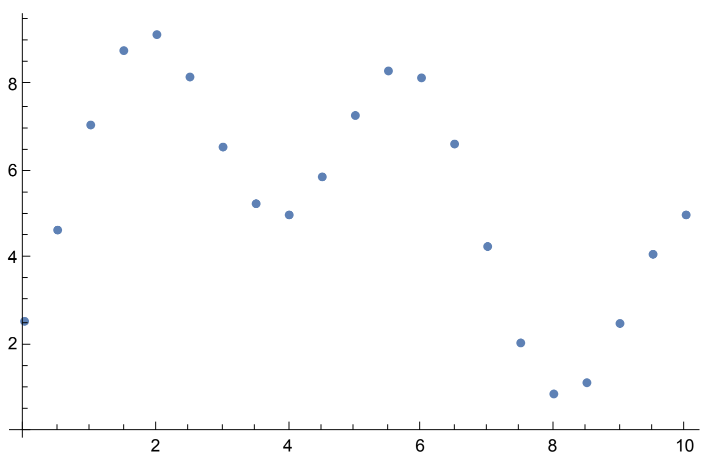
time	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
height	2.54	4.65	7.08	8.80	9.17	8.19	6.57	5.26	5.00	5.88	7.30

time	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
height	8.33	8.17	6.64	4.27	2.04	0.86	1.12	2.49	4.09	5.00

```
In[∘]:= {time, height} = {{0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0,
    7.5, 8.0, 8.5, 9.0, 9.5, 10.0}, {2.54, 4.65, 7.08, 8.80, 9.17, 8.19, 6.57, 5.26,
    5.00, 5.88, 7.30, 8.33, 8.17, 6.64, 4.27, 2.04, 0.86, 1.12, 2.49, 4.09, 5.00}}
ListPlot[Transpose[{time, height}]]

Out[∘]= {{0, 0.5, 1., 1.5, 2., 2.5, 3., 3.5, 4., 4.5, 5., 5.5, 6., 6.5, 7., 7.5, 8., 8.5, 9., 9.5, 10.},
  {2.54, 4.65, 7.08, 8.8, 9.17, 8.19, 6.57, 5.26, 5., 5.88,
  7.3, 8.33, 8.17, 6.64, 4.27, 2.04, 0.86, 1.12, 2.49, 4.09, 5.}}
```

Out[∘]=



1.1. Estimate the greatest distance from the floor during this time. Approximately at what times is the maximal distance reached?

It appears that the greatest distance from the floor is at around t=2, with a height of 9.17 feet.

1.2. Estimate the smallest distance from the floor during this time. Approximately at what times is the minimal distance reached?

The minimal height is at 8 seconds, with a height of 0.86 feet.

1.3. Plot the points. Assuming that the motion is periodic, and that you have witnessed at least one period, what is your estimate for the period of the motion? How many local (non-global) extrema might you identify?

Since we don't see any exact repetition, the period has to be 10 seconds. It the motion is "nearly periodic", then it appears that we might be seeing a repeat of the first few half seconds at the end -- in which case the period might be about 9 seconds.

If the former, then there are 3 local maxes, and 3 local mins; if the latter, then I'd say that there are 2 of each.

2. The following questions all refer to the graph $y = f(x)$ given below. Estimate your results from the graph.

In[20]:=
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f[x_] := 30 (CubeRoot[(x + 3)^2] - 2)/(1 + x^2)
f[0.0]
f[5.0]
Show[Graphics[{Thickness[0.003],
   Table[{Line[{{n, -10}, {n, 10}}], Line[{{-10, n}, {10, n}}]}, {n, -10, 10, 5}]}],
  Plot[30 (CubeRoot[(x + 3)^2] - 2)/(1 + x^2), {x, -10, 10}], PlotRange → {-10, 10},
  GridLines → {Range[-10, 10], Range[-10, 10]}, AspectRatio → Automatic,
  PlotLabel → HoldForm[y = f[x]], BaseStyle → FontSize → 14, Axes → True, AxesStyle → Thick]
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Out[21]= 2.40251

Out[22]= 2.30769

Out[23]=

2.1. On the closed and bounded interval [0, 5].

2.1.1. What are the critical numbers inside this interval?

The derivative is 0 or undefined at the points (maybe at) 0 -- depends on whether the slope gets infinitely steep); and maybe at about 0.8.

2.1.2. What is the maximum value of $f(x)$? For what values of x is this maximum value attained?
At about x = 0.8. The maximum value is about 8.

2.1.3. What is the minimum value of $f(x)$? For what values of x is this minimum value attained?

It’s a little tricky to see if the value at 0 is smaller than the value at x=5 -- but I think so. and it appears to be about 2.

2.2. On the closed and bounded interval [−6, 0].

2.2.1. What are the critical numbers inside this interval?

The derivative is 0 or undefined at the points x=-3 and at x= -1.2 (or so).

2.2.2. What is the maximum value of $f(x)$? For what values of x is this maximum value attained?
The function achieves a maximum at 0, with value of f(0) (which may be around 2).

2.2.3. What is the minimum value of $f(x)$? For what values of x is this minimum value attained?

It is a minimum at x = -1.2, with a value of about -6.4.

3. Let $g(x) = \frac{x^2+3x}{x^2+x+2} + 5$.

In[∘]:=
```
g[x_] := (x^2 + 3 x) / (x^2 + x + 2) + 5
-2 * Simplify[g'[x] / (-2)]
```

Out[∘]=
$$-\frac{2\left(-3-2x+x^2\right)}{\left(2+x+x^2\right)^2}$$

3.1. Show that $g'(x) = -2\,\frac{(x+1)(x-3)}{(x^2+x+2)^2}$.

$-\frac{2\left(-3-2x+x^2\right)}{\left(2+x+x^2\right)^2}$ has roots -1 and 3. So these are the critical numbers (the denominator is never 0). Since this is a smooth function, these are the only two critical numbers.

In[∘]:=
```
{g[0], g[3], g[5]} * 1.0
```

Out[∘]=
```
{5., 6.28571, 6.25}
```

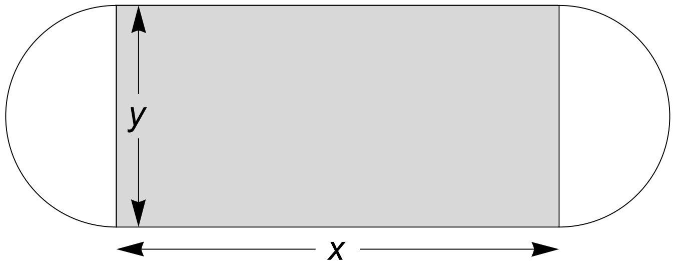
3.2. For x-values in the interval [0, 5], what is the maximum value of $g(x)$? For what values of x is this maximum attained?

On this interval, closed and bounded, the max must occur at a critical point or at the boundary. It’s a little bigger at x = 3.

3.3. For x-values in the interval [0, 5], what is the minimum value of $g(x)$? For what values of x is this minimum attained?

The global minimum on this interval occurs at x=0, with a value of 5.

4. A local high school is designing a new sports field. Its shape is rectangular with two semicircles at the opposite sides.



The field needs to have a 400 meter track around its perimeter. What are the dimensions x and y that make the area of the field as large as possible while maintaining the 400 meter perimeter?

In[54]:=
```
Clear[area, x, y]
area[x_, y_] := x y
perimeter[x_, y_] := 2 x + 2 Pi (y / 2)
Solve[perimeter[x, y] == 400, y]
y[x_] := (200 - x) * 2 / Pi (* 2x+2 Pi (y/2) = 400  constraint equation *)
Simplify[area[x, y[x]]]
dAdx = D[area[x, y[x]], x]
soln = Solve[dAdx == 0, x]
xmax = x /. soln[[1]][[1]]
ymax = y[xmax]
N[%]
Plot[area[x, y[x]], {x, 0, 200}]
```

Out[57]= $\left\{\left\{y \to -\frac{2(-200+x)}{\pi}\right\}\right\}$

Out[59]= $-\frac{2(-200+x)\,x}{\pi}$

Out[60]= $\frac{2(200-x)}{\pi} - \frac{2x}{\pi}$

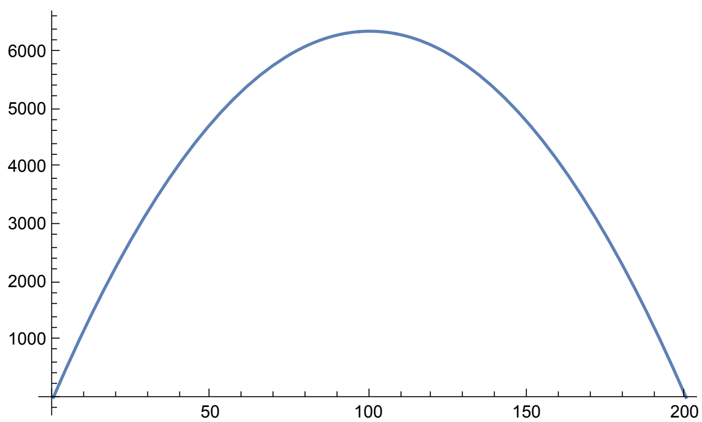
Out[61]= $\{\{x \to 100\}\}$

Out[62]= 100

Out[63]= $\frac{200}{\pi}$

Out[64]= 63.662

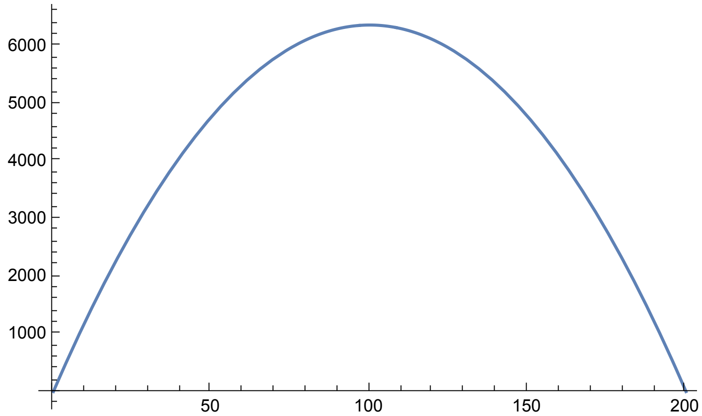
Out[65]=



{{x → 100}}

y

Out[32]=



y = 200/Pi approx 63.6 meters, x=100meters: perfect for football and soccer!:)