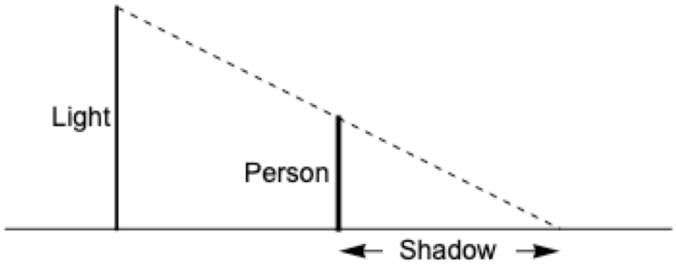
3.5: Related Rates Worksheet

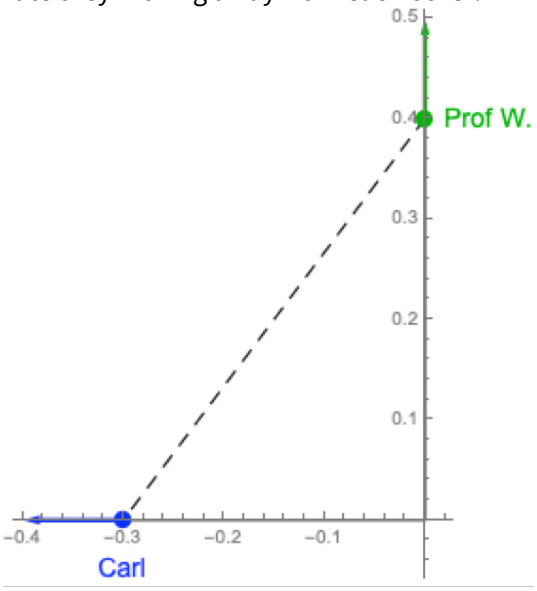
1. A 6-foot person is out walking at night. As they walk away from a 17-foot tall street lamp, the length of their shadow increases. At one point they are 14 feet from the street lamp and walking 4.5 feet per second. How fast is their shadow increasing in length?

A sketch of this situation is helpful.



Similar triangles showing a person walking near a street light.

2. Professor Wilkinson is driving north on US 27 while his son Carl is driving west on the AA highway. At 1:00 PM when Professor Wilkinson is 0.4 miles past the US 27 overpass of the AA highway, Carl is 0.3 miles also past that point. At that time Professor Wilkinson's speedometer reads 45 mph and Carl's reads 55 mph. We want to find the rate they moving away from each other.



3. Activity 3.5.2

Activity 3.5.2. A water tank has the shape of an inverted circular cone (point down) with a base of radius 6 feet and a depth of 8 feet. Suppose that water is being pumped into the tank at a constant instantaneous rate of 4 cubic feet per minute.

a. Draw a picture of the conical tank, including a sketch of the water level at a point in time when the tank is not yet full. Introduce variables that measure the radius of the water's surface and the water's depth in the tank, and label them on your figure.

b. Say that r is the radius and h the depth of the water at a given time, t. What equation relates the radius and height of the water, and why?

c. Determine an equation that relates the volume of water in the tank at time t to the depth h of the water at that time.

d. Through differentiation, find an equation that relates the instantaneous rate of change of water volume with respect to time to the instantaneous rate of change of water depth at time t.

e. Find the instantaneous rate at which the water level is rising when the water in the tank is 3 feet deep.

f. When is the water rising most rapidly: at $h = 3$, $h = 4$, or $h = 5$?

The volume of a cone of radius r and height h is

$$V = {}^{1}/_{3}\,\pi\, r^2\, h$$