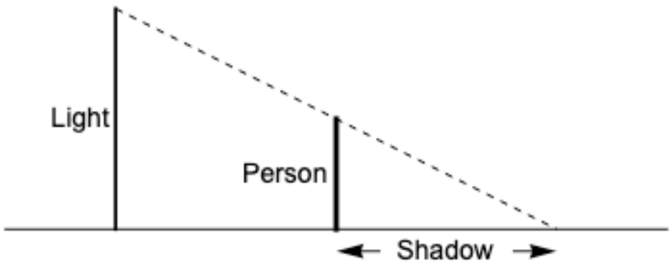
3.5: Related Rates Worksheet

1. A 6-foot person is out walking at night. As they walk away from a 17-foot tall street lamp, the length of their shadow increases. At one point they are 14 feet from the street lamp and walking 4.5 feet per second. How fast is their shadow increasing in length?

A sketch of this situation is helpful.



Similar triangles showing a person walking near a street light.

This is the same as the example we did in class -- different height of lamppost, is all. Again, similar triangles is the way to go.

So (x+s)/17=s/6,

which we can solve for s = 6/11*x

This is linear, so ds/dt = 6/11 dx/dt; hence, when the person is walking at 4.5 ft/s, the shadow is moving at

6/11*9/2 = 27/11 ft/s

or approximately 2.45455 ft/s. Notice that the 14 feet was not useful; the answer is independent of the position of the person.

In[20]:=
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Solve[(x + s) / 17 == s / 6, s]
6.0 / 11 * 9 / 2
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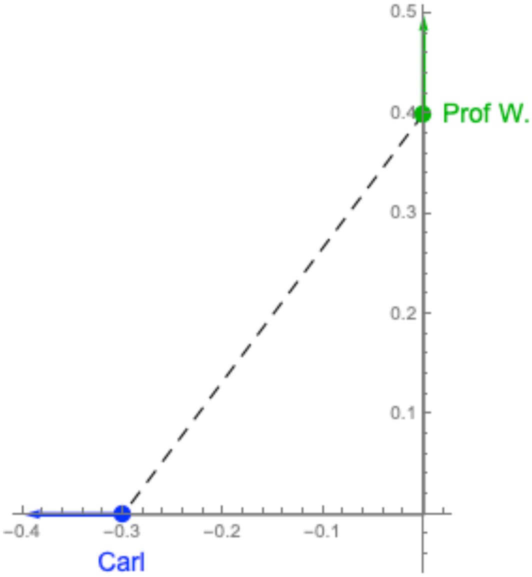
Out[20]= $\left\{\left\{s \to \frac{6\,x}{11}\right\}\right\}$

Out[21]= 2.45455

2. Professor Wilkinson is driving north on US 27 while his son Carl is driving west on the AA highway. At 1:00 PM when Professor Wilkinson is 0.4 miles past the US 27 overpass of the AA highway, Carl is 0.3 miles also past that point. At that time Professor Wilkinson’s speedometer reads 45 mph and Carl’s reads 55 mph. We want to find the rate they moving away from each other.



We are going to relate the distance between the two via their rates dx/dt and dy/dt. So we need an equation relating distance and x and y -- which Pythagoras gives us:

d = Sqrt[x^2+y^2]

Then we use implicit differentiation to find dd/dt:

$$dd/dt = \frac{x[t]\, x'[t] + y[t]\, y'[t]}{\sqrt{x[t]^2 + y[t]^2}}$$

So when dx/dt=-55, x = 0.3, dy/dt=45, y=0.4,

x = -0.3

dd/dt=69mph

In[34]:=
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Clear[x, x', y, y']
relatedRates = Simplify[D[Sqrt[x[t]^2 + y[t]^2], t]]
{x[t], x'[t], y[t], y'[t]} = {-0.3, -55, 0.4, 45}
relatedRates
Clear[x, x', y, y']
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Out[35]= $\frac{-55\, x[t] + 45\, y[t]}{\sqrt{x[t]^2 + y[t]^2}}$

Out[36]= {-0.3, -55, 0.4, 45}

Out[37]= 69.

3. Activity 3.5.2

Activity 3.5.2. A water tank has the shape of an inverted circular cone (point down) with a base of radius 6 feet and a depth of 8 feet. Suppose that water is being pumped into the tank at a constant instantaneous rate of 4 cubic feet per minute.

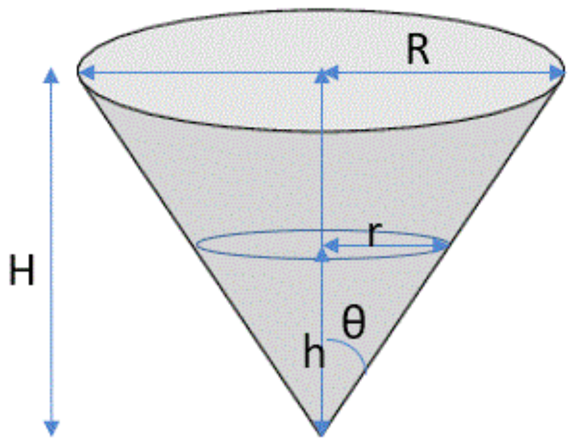
a. Draw a picture of the conical tank, including a sketch of the water level at a point in time when the tank is not yet full. Introduce variables that measure the radius of the water's surface and the water's depth in the tank, and label them on your figure.

b. Say that *r* is the radius and *h* the depth of the water at a given time, *t*

The volume of a cone of radius r and height h is

$$V = {}^{1}/_{3}\, \pi\, r^2\, h$$

This seems like the tricky one, because of the set up. We’ve got an inverted conical tank, which sounds complicated (but isn’t really so bad).



b. Again, similar triangles are useful: R=6, H=8; so r/6=h/8;

c. I gave this one to you: V = 1/3 pi r^2 h, so we need to replace r by r(h):

V = 1/3 pi (3/4h)^2 h = 3/16 pi h^3

d. dV/dt=3/16 pi 3 h^2 dh/dt; therefore
dh/dt = dV/dt 16/9 (1/(pi h^2))

e. Since dV/dt=4 ft^3/minute,

dh/dt = 64/9(1/(pi 9)) = 64/81/(pi) ft/min,
approximately
0.25 ft/min

f. This equation for the rate dh/dt is a decreasing function of h; hence, it is larger when h is smaller.

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In[39]:= 64 / (81 * 3.1415926)
Out[39]= 0.251504
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