Chain Rule Worksheet

1. Find the period and the derivative for the following sinusoidal functions.

a. $\cos(x)$
Period: 2Pi
Derivative: -sin(x)

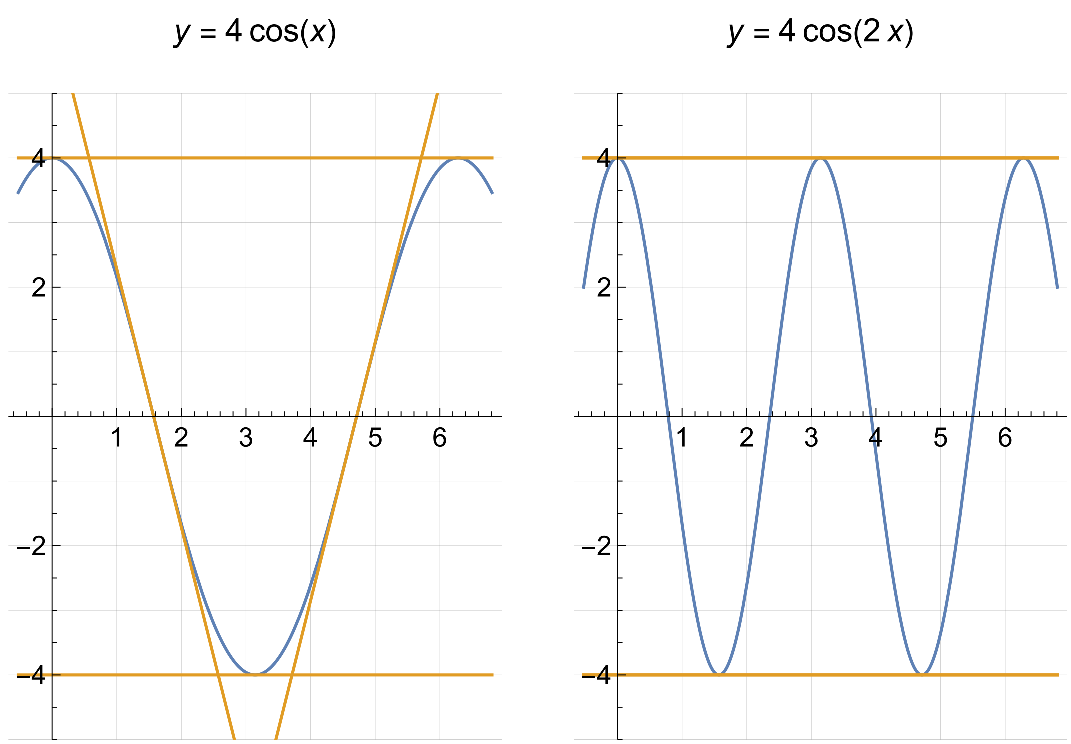
b. $3\cos(2x)$
Period: Pi
Derivative: -6 sin(2x)

c. $\cos\left(\frac{x}{2}\right)+5$
Period: 4Pi
Derivative: $-\sin\left(\frac{x}{2}\right)/2$

d. $-6\cos(4x)+2$
Period: Pi/2
Derivative: 24 sin(4x)

2. Below are the graphs of $f(x)=4\cos(x)$ and $g(x)=4\cos(2x)$. On those graphs, draw the tangent lines at the indicated x-values and estimate the slopes to get the derivatives.

Out[3317]=



x	$(\cos(x))'$ as slope	$(\cos(2x))'$ as slope
0		
$\frac{\pi}{2} \approx 1.6$		
$\pi \approx 3.1$		
$\frac{3\pi}{2} \approx 4.7$		
$2\pi \approx 6.3$		

Here are the values your estimates should be close to in your table:

In[3318]:=
```
MatrixForm[Table[{x, fx'[x], f2x'[x]}, {x, 0, 2 Pi, Pi / 2}]]
```

Out[3318]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ \frac{\pi}{2} & -4 & 0 \\ \pi & 0 & 0 \\ \frac{3\pi}{2} & 4 & 0 \\ 2\pi & 0 & 0 \end{pmatrix}$$

3. Consider $y = e^{\sin(x)} + 1$ at $x = 0$.

In[3319]:=
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f[x_] := E^Sin[x] + 1
```

3.1. Compute the derivative y'(x).

In[3320]:=
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f'[x]
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Out[3320]= $e^{\text{Sin}[x]}$ Cos[x]

3.2. Find an equation for the tangent line to $y = e^{\sin(x)} + 1$ at $x = 0$.

In[3321]:=
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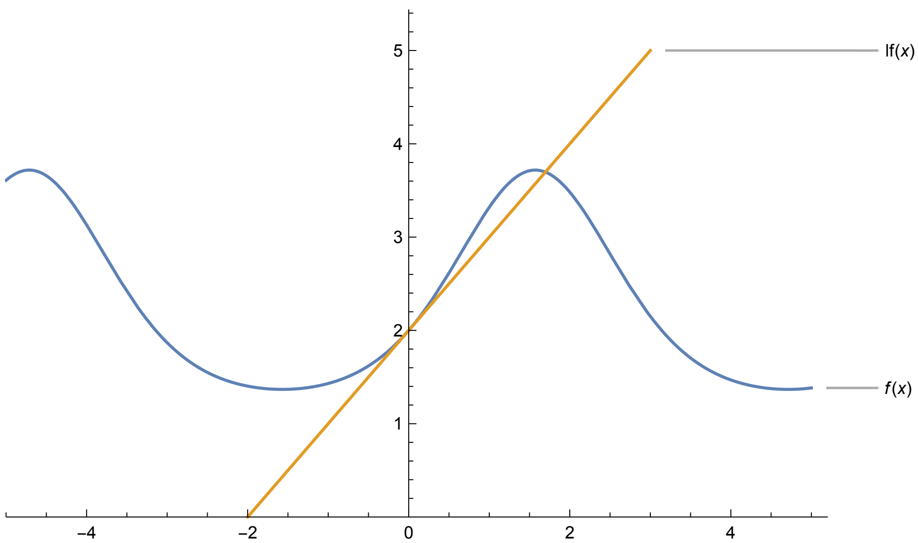
lf[x_] = f[0] + f'[0] (x - 0)
```

Out[3321]= 2 + x

3.3. In Desmos or on a graphing calculator plot both $y = e^{\sin(x)} + 1$ and the tangent line you found. Sketch the results below.

Out[3322]=



4. One claims that the function $f(x) = 3\cos\left(\frac{2\pi}{365}(x - 171)\right) + 12$ gives a pretty good approximation for the hours of daylight for day x of the year.

In[3323]:=
```
f[x_] := 3 Cos[2 Pi / 365 * (x - 171)] + 12
```

4.1. Today is day 76 of the year. According to this model, how many hours and minutes of daylight should we expect? Do a web search to find out how well the model is working here in Cincinnati (if it's not doing well, how might you fix it?).

In[3324]:=
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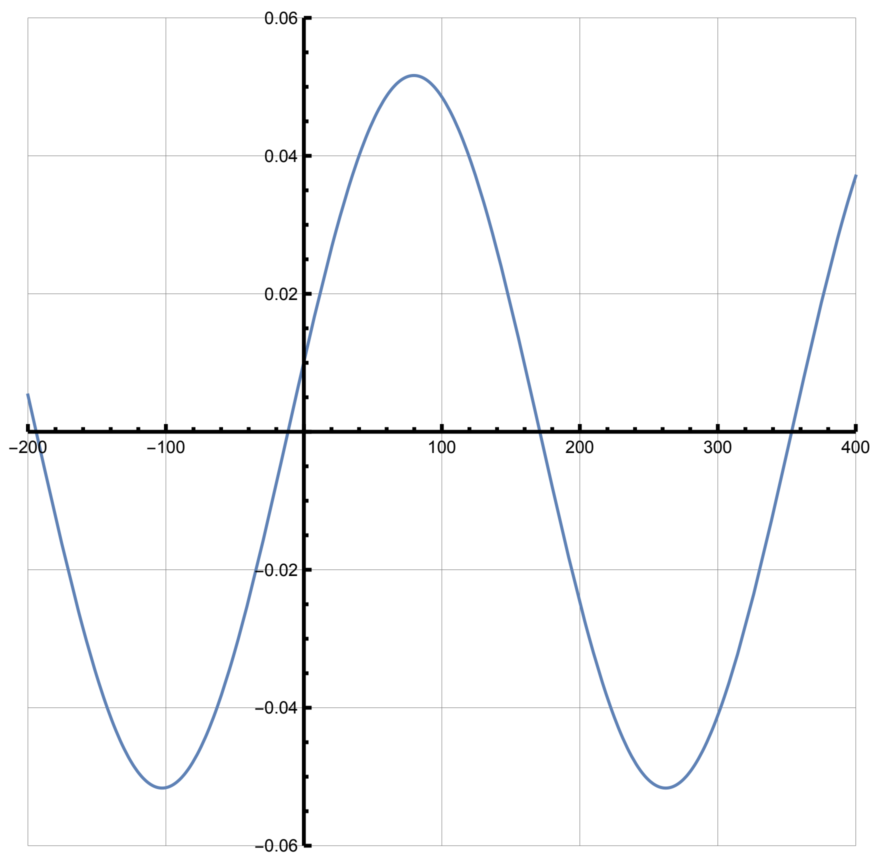
f[76.0]
```

Out[3324]=
```
11.8064746516521
```

According to https://www.timeanddate.com/sun/usa/cincinnati?month=3, it was almost exactly 12 hours (11:58) on this date. So we're a little below. Seems like a slight shift might be in order.

4.2. Find $f'(x)$, and plot it.

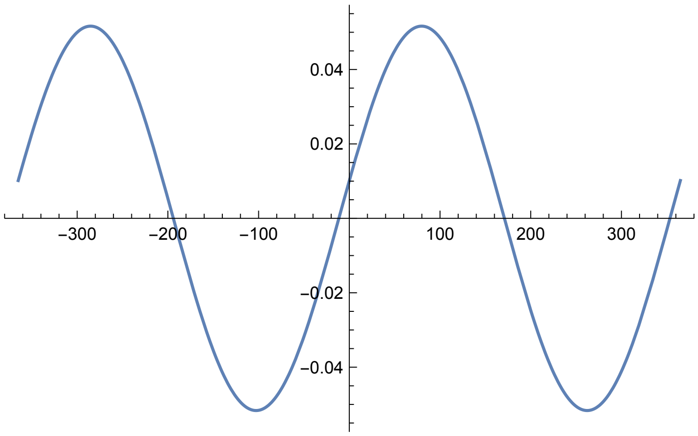
Out[3325]=



Out[3327]= 11.8064746516521

Out[3328]= 0.0515350556953299

Out[3329]=



4.3. You should find that $f'(76) = 0.0515350556953299$: what does the sign and magnitude indicate about today's daylight?

That the number of hours of daylight is increasing (sign positive), and that it's nearly at its apex.

5. Consider the graph $y = \sqrt{x^4 - 3x^2 + 5}$.

In[3330]:=
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f[x_] := Sqrt[x^4 - 3 x^2 + 5]
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5.1. What is $\frac{dy}{dx}$?

In[3331]:=
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f'[x]
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Out[3331]=
$$\frac{-6x + 4x^3}{2\sqrt{5 - 3x^2 + x^4}}$$

5.2. Using the derivative, find all the x-values where the graph has horizontal tangents.

In[3332]:=
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Solve[f'[x] == 0, x]
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Out[3332]=
$$\left\{\{x \to 0\}, \left\{x \to -\sqrt{\frac{3}{2}}\right\}, \left\{x \to \sqrt{\frac{3}{2}}\right\}\right\}$$

5.3. Graph this function in Desmos or on a graphing calculator. Sketch the results below and indicate on the graph the points with horizontal tangents. Does it agree with your calculations?

Out[3334]=