

Last quiz, you proved the predicate wff


$$(\forall y) [Q(x, y) \rightarrow P(x)] \rightarrow [(\exists y) Q(x, y) \rightarrow P(x)]$$

Every student who proved it did so directly.

1. (6 pts) Prove it by contraposition.

$$[(\exists y) Q(x, y) \rightarrow P(x)]' \rightarrow (\forall y) [Q(x, y) \rightarrow P(x)]'$$


$$[(\forall y) Q(x, y) \wedge P(x)]' \rightarrow (\exists y) [Q(x, y) \wedge P(x)]'$$

1) $(\forall y) Q(x, y)$ [The] 

2) $P(x)'$ [The]

3) $Q(x, y)$ [1, $\forall i$]

4) ~~$Q(x, y) \wedge P(x)'$~~ []

4.5) $Q(x, y) \wedge P(x)'$ [2, 3, Add] ^(con) 

5) $(\exists y) [Q(x, y) \wedge P(x)']$ [4, $\exists y$]

2. (4 pts) Prove it by contradiction.

$$(\forall v) [Q(x, v) \rightarrow P(x)] \rightarrow [(\exists v) Q(x, v) \rightarrow P(x)]$$

1. $(\forall v) [Q(x, v) \rightarrow P(x)]$ hyp

2. $(\exists v) [Q(x, v)]$

3. $P'(x)$ Contradiction

4. $Q(x, z)$ ei 2

5. $\neg [Q(x, z) \rightarrow P(x)]$ vi 1

6. $Q(x, z)$ 3 5 mt

7. ~~$(\forall v) [Q(x, v)]$~~ 6 ug
 $Q(x, z) \wedge \neg Q(x, z)$ inconsistent,

7 and 4 cant both be right
therefore proven by contradiction



Nice
work

2. (4 pts) Prove it by contradiction.

1. $\forall y (Q(x, y) \rightarrow P(x))$ hyp

2. $(\exists y (Q(x, y))) \rightarrow P(x)$ hyp

3. $\forall P(x) \rightarrow \dots$ hyp CONTRADICTION

4. $Q(x, b) \rightarrow P(x)$ 2, ei

5. $Q(x, b) \rightarrow P(x)$ 3, 4i

6. $Q(x, b) \wedge Q(x, b)'$ 3, 4, MT

7. $Q(x, b) \wedge Q(x, b)'$ 4, 6, conj

Using a contradictory hyp $P(x)'$, $Q(x, b) \wedge Q(x, b)'$ which is a contradiction.

Well done.