Hilbert's Hotel

Where did the guests go?

One couple shows up, & we found them a room:

One-to-one correspondance

$\mathbb{N}$		S
1	$\longleftrightarrow$	
2	$\longleftrightarrow$	
3	$\longleftrightarrow$	
4	$\longleftrightarrow$	
$\vdots$		

Rule: $n \rightarrow$

$\text{Card}(\mathbb{N}) = \text{Card}(S)$

Infinite school bus shows up:

$\mathbb{N}$		S
1	$\longleftrightarrow$	
2	$\longleftrightarrow$	
3	$\longleftrightarrow$	
4	$\longleftrightarrow$	
$\vdots$		

Rule: $n \rightarrow$

$\text{Card}(\mathbb{N}) = \text{Card}(S)$

Infinitely many infinite school buses show up!

$\mathbb{N}$		S
1	$\longleftrightarrow$	
2	$\longleftrightarrow$	
3	$\longleftrightarrow$	
4	$\longleftrightarrow$	
$\vdots$		

Rule: $n \rightarrow$

Power sets are bigger!

Set S	# of elements	Power set P(S)	# of elements
$\{ \}$	0		
$\{1\}$	1		
$\{1,2\}$	2		
$\{1,2,3\}$	3		

Imagine that there is a one-to-one correspondance between the set $S = \{1,2\}$ & its power set $P(S)$. Here it is:

S		$P(S)$
1	$\longleftrightarrow$	$\{1\}$
2	$\longleftrightarrow$	$\{ \}$

Oops! $A = \{2\}$ is not on the list:

If $1 \in \{1\}$, $1 \notin A$; otherwise $1 \in A$.

If $2 \in \{ \}$, $2 \notin A$; otherwise $2 \in A$.

We can disprove this supposed one-to-one correspondance by producing an element of $P(S)$ that is not on the list above; that isn't at the dance. This is called "Proof by contradiction".