LESSON

1

Symmetry

The ink drawing shown here was created by the psychologist Roger Shepard. Shepard titled it "Reflecting Prince" and wrote that it "can be seen either as a frontal view of a strangely wide-faced (perhaps extraterrestrial) being or, alternatively, as a profile view of a normally proportioned human prince with his face (and hand) . . . pressed against a mirror."*

If you place a mirror perpendicular to the page so that the edge of the mirror lies on the vertical line down the center of this figure, you will see that one side of the picture is indeed a reflection of the other. This center line is called a mirror line, and it is the line of symmetry of the figure. Shepard's drawing can be seen as a single face because the human face itself has *line symmetry*.

Mind Sights, by Roger N. Shepard (W. H. Freeman, 1990).

A figure has **line symmetry** if there is a line along which a mirror can be placed to reflect either half of the figure so that it reproduces the other half.

Another way to see if a figure has line symmetry is to fold the figure along the supposed line to see if the two halves coincide (fit together exactly). Look, for example, at the flag of Turkey shown here. It is symmetrical with respect to a horizontal line through its center. If the flag is folded along this line, the pattern on one side of the fold coincides with the pattern on the other side.

A figure can be symmetrical with respect to more than one line. A square, for example, has four lines of symmetry. It is easy to see that the horizontal and vertical lines in the figure below are lines of symmetry; the two lines through the opposite corners of the square are less obvious.
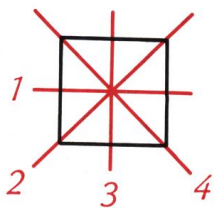

The paper windmill shown here is symmetrical, yet it does not have any lines of symmetry. It is impossible to fold it along a line so that any two halves coincide. The windmill can be *rotated*, however, into other positions that look exactly the same. For example, it can be rotated 90° (through one-quarter of a circle) and look exactly the same. Therefore, it has *rotational symmetry*.
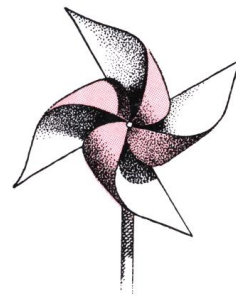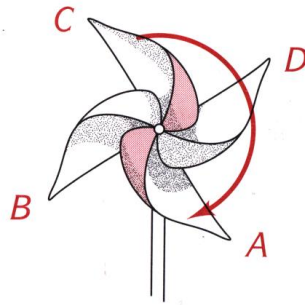

A figure has **rotational symmetry** if it can be rotated through an angle of less than 360° so that it coincides with its original position.
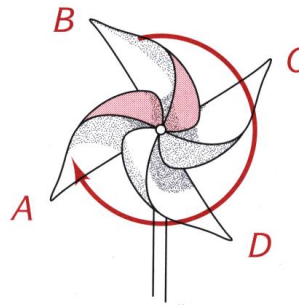
The point about which the figure is rotated is called the *axis* of symmetry. The axis of symmetry of the paper windmill is the pin about which it turns.
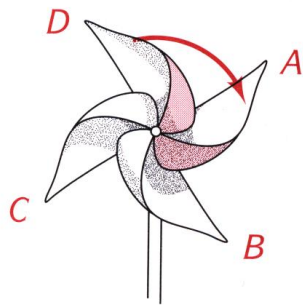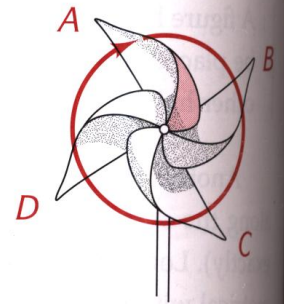
One-quarter circle (90°)

One-half circle (180°)

Three-quarters circle (270°)

Full circle (360°)

Notice that the windmill can be turned through four different angles up to and including 360° so that it looks exactly the same: 90°, 180°, 270°, and 360°.*

For this reason, the windmill is said to have "4-fold" rotational symmetry.

EXERCISES

SET I

Someone was a bit careless in painting the word ONLY on this freeway off-ramp in Los Angeles. The stencil must have accidentally been turned over so that left and right were reversed when the N was painted.
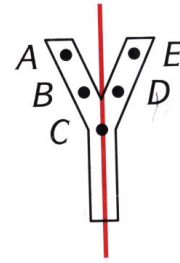
1. For which of the letters in ONLY would this have not mattered?

© 1991, Alfred B. MacInnes, Pittsfield (MA) H. S., '51

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*Angles and their measures are discussed on pages 656–657.

The letter Y has symmetry with respect to a vertical line, as this figure shows. If a mirror were placed on the line, which point would be the reflection of
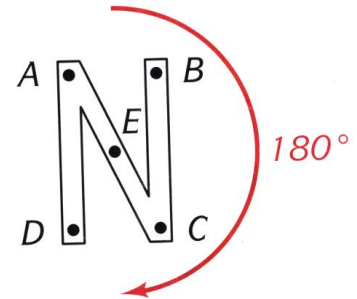
2. point A?   3. point D?   4. point C?

In general, if a figure has line symmetry, exactly where is the reflection of a point that

5. is on the line?   6. is not on the line?

The letter N has rotational symmetry because it looks the same if it is rotated 180°. If the letter N is rotated in this way, to which point does

7. point A move?   8. point D move?   9. point E move?

10. In general, if a figure has rotational symmetry, which point of the figure does not move?

11. Which letter in ONLY has both line symmetry and rotational symmetry?

A reflection of the horse and its rider is clearly visible in this photograph.

12. What is acting as a mirror?

13. Does the part of the picture showing the horse and rider and their reflection appear to have line symmetry? (Ignore the rest of the picture.)

14. Does it have rotational symmetry? Explain why or why not.

Photograph by Joe Vitti

Many playing cards are symmetrical. The two cards shown here are examples.
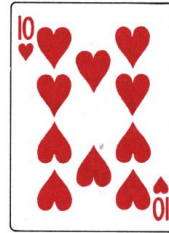
15. What kind of symmetry do they have?

16. Why do you suppose they have this symmetry?

The star in a circle has been the symbol of Mercedes-Benz cars since 1909.

17. How many lines of symmetry does this symbol have?
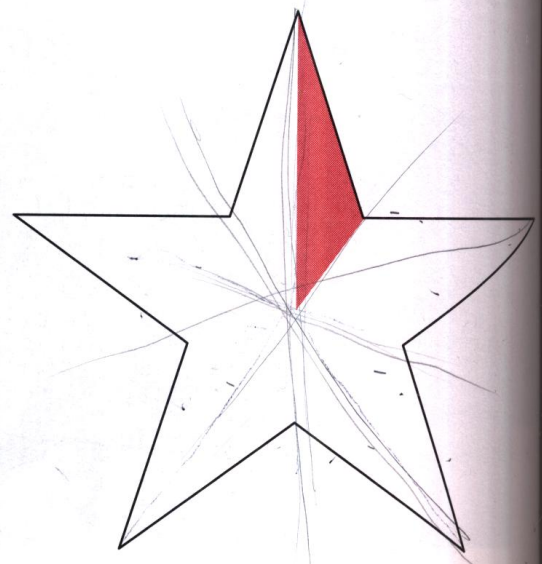
The symbol has 3-fold rotational symmetry.

18. What does this mean?

19. To have rotational symmetry, must a figure look the same upside down?

Mercedes Benz symbol

A starfish has many symmetries. Trace the star in the drawing to the right of the photograph on tracing paper. Also copy the shaded part on your drawing.

20. In how many ways can you place your tracing on this figure so that the whole star fits the whole star but the shaded part
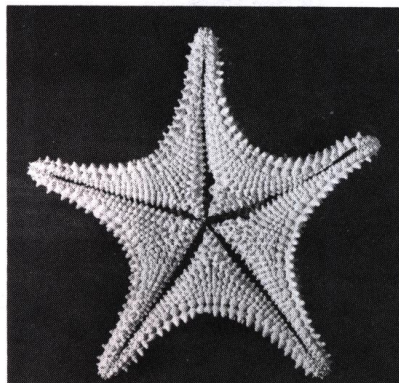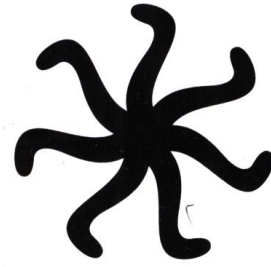

Courtesy of Barbara Ferenstein

is in a different place? (Count the original position of the tracing as one of the ways.)

21. How many lines of symmetry does the star have?

22. What kind of rotational symmetry does it have?

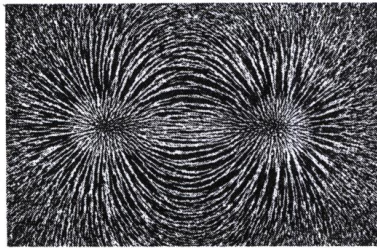This "running-legs" symbol has 7-fold rotational symmetry but no lines of symmetry.

Describe, as completely as you can, the symmetry, if any, possessed by each of the following figures.
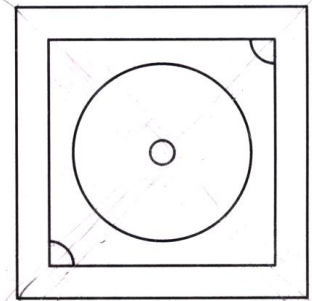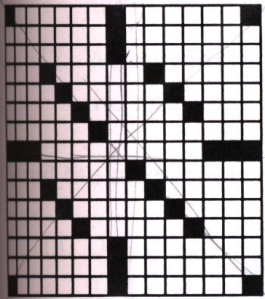
23.

©Scott Kim, 1989

24.

Magnet with iron filings

25.
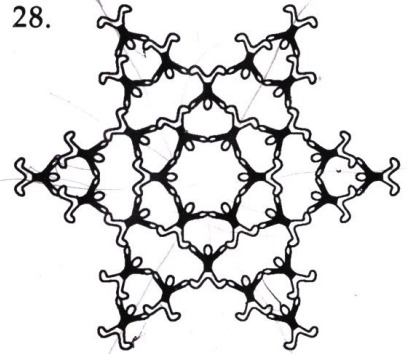
Floor plan of wrestling ring

26.

Crossword puzzle diagram

27.

© 1991 Robert Petrick

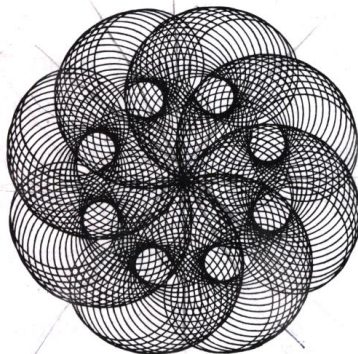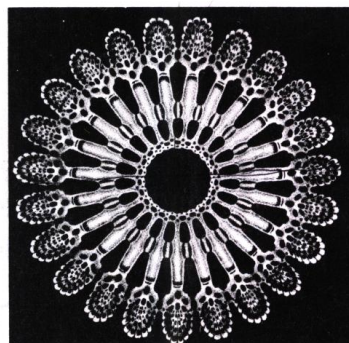28.

Skydivers in formation freefall

29.

Figure by Y. Chernikhov

30.

Sea urchin
From *Art Forms in Nature*, by Ernst Haeckel,
Dover Publications, Inc., New York, 1974.

SET III

This photograph was taken as a gag.

Photograph by Peter Renz

1. How might the scene be changed to make it more convincing?
2. Explain by means of symmetry why the board would still look wrong, even with this change.

One part of the photograph is puzzling.

3. What is it that is puzzling?
4. Explain how it could look that way.