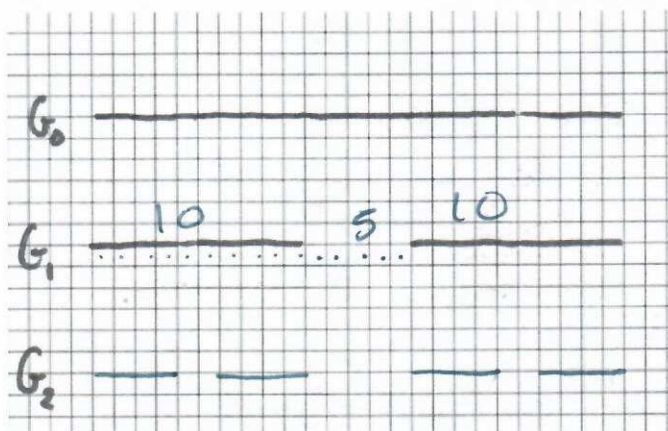


**Directions:** Show your work! Answers without justification will likely result in few points, and doesn't allow me to give partial credit. Indicate clearly your answer to each problem (e.g., put a box around it). Good luck! And remember to send me your knot photo....

**Problem 1: (25 pts) Fractals**

- a. Consider this stick of (red) licorice of length 1: at each generation, one nibbles the middle **fifth** from each stick of licorice in the previous generation.



25  
2/5 1/5 2/5



- i. (10 pts) Add the next stage of the fractal (generation 2, or  $G_2$ ) to the figure above.  
ii. (3 pts) How is the fractal's total length changing from step to step?

$$G_0 - 1 - 4/5^0$$

$$G_1 - 4/5 = 4/5^1$$

$$G_2 - (4/5)^2$$



- iii. (3 pts) How is the number of sticks increasing with each round?

$$G_0 - 1 - 2^0$$

$$G_1 - 2 - 2^1$$

$$G_2 - 4 - 2^2$$

In exponents of 2



- iv. (2 pts) How many sticks will there be at generation  $G_6$ ?

$$2^6 = 64$$



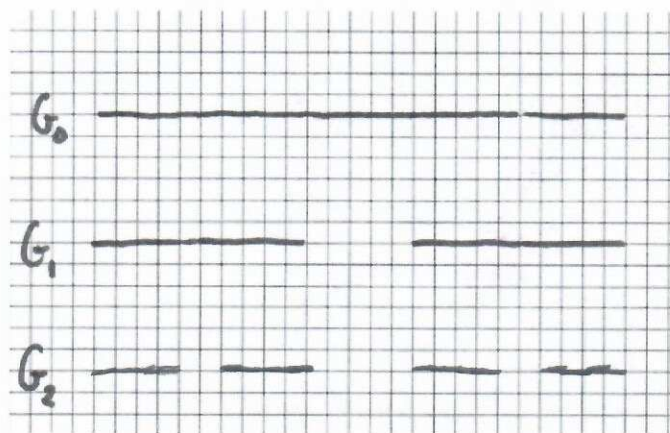
- v. (2 pts) What will be the length of licorice at generation  $G_6$ ?

$$(4/5)^6$$



**Problem 1: (25 pts) Fractals**

- a. Consider this stick of (red) licorice of length 1: at each generation, one nibbles the middle fifth from each stick of licorice in the previous generation.



- i. (10 pts) Add the next stage of the fractal (generation 2, or  $G_2$ ) to the figure above.  
 ii. (3 pts) How is the fractal's total length changing from step to step?

$$G_0 \left(\frac{4}{5}\right)^0$$

$$G_1 \left(\frac{4}{5}\right)^1$$

$$G_2 \left(\frac{4}{5}\right)^2$$

It loses a fifth of each remaining stick in each generation

- iii. (3 pts) How is the number of sticks increasing with each round?

The number of sticks in each generation is double the number in the previous generation

- iv. (2 pts) How many sticks will there be at generation  $G_6$ ?

$G_0$	1	$G_3$	8
$G_1$	2	$G_4$	16
$G_2$	4	$G_5$	32

$G_6$  64

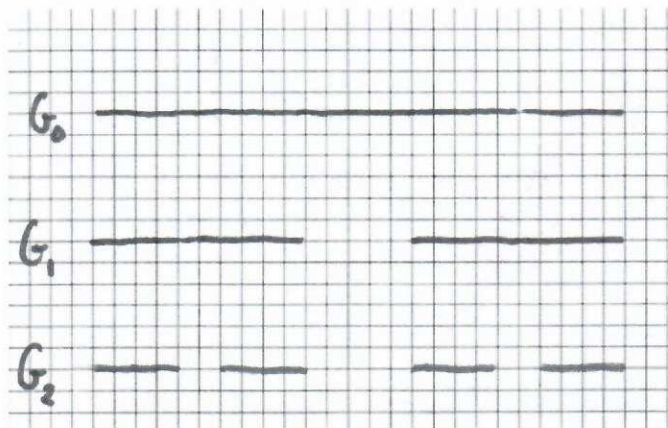
- v. (2 pts) What will be the length of licorice at generation  $G_6$ ?

$$\left(\frac{4}{5}\right)^6$$

0.262144

**Problem 1: (25 pts) Fractals**

- a. Consider this stick of (red) licorice of length 1: at each generation, one nibbles the middle **fifth** from each stick of licorice in the previous generation.



- i. (10 pts) Add the next stage of the fractal (generation 2, or  $G_2$ ) to the figure above.  
 ii. (3 pts) How is the fractal's total length changing from step to step?

$$G_0: \left(\frac{4}{5}\right)^0 = 1$$

$$G_1: \left(\frac{4}{5}\right)^1 = \frac{4}{5}$$

$$G_2: \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

- iii. (3 pts) How is the number of sticks increasing with each round?

$$G_0: 1 = 2^0$$

$$G_1: 2 = 2^1$$

$$G_2: 4 = 2^2$$

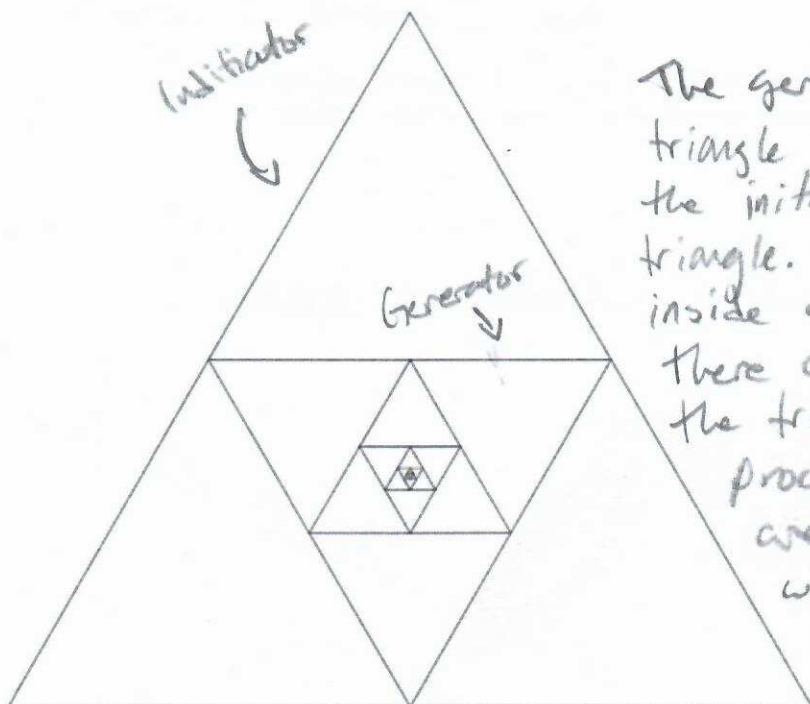
- iv. (2 pts) How many sticks will there be at generation  $G_6$ ?

$$G_6: 2^6 = 64$$

- v. (2 pts) What will be the length of licorice at generation  $G_6$ ?

$$G_6: \left(\frac{4}{5}\right)^6 = \frac{4096}{15625}$$

b. (5 pts) Start with an equilateral triangle. After many generations, the fractal looks like this:



The generator is the second triangle that we are creating after the initiator which is the biggest triangle. The generator is placed inside of the triangle in which there are 4 total triangles within the triangle. We then repeat that process where each time we are creating 4 triangles within the previous one and the size is getting smaller as we go.

In the space above, describe the process by which this fractal is created: what is the generator; what does one do at each step?

**Problem 2: (25 pts) Platonic Solids**

a. (6 pts) Fill in the missing entries in this table:

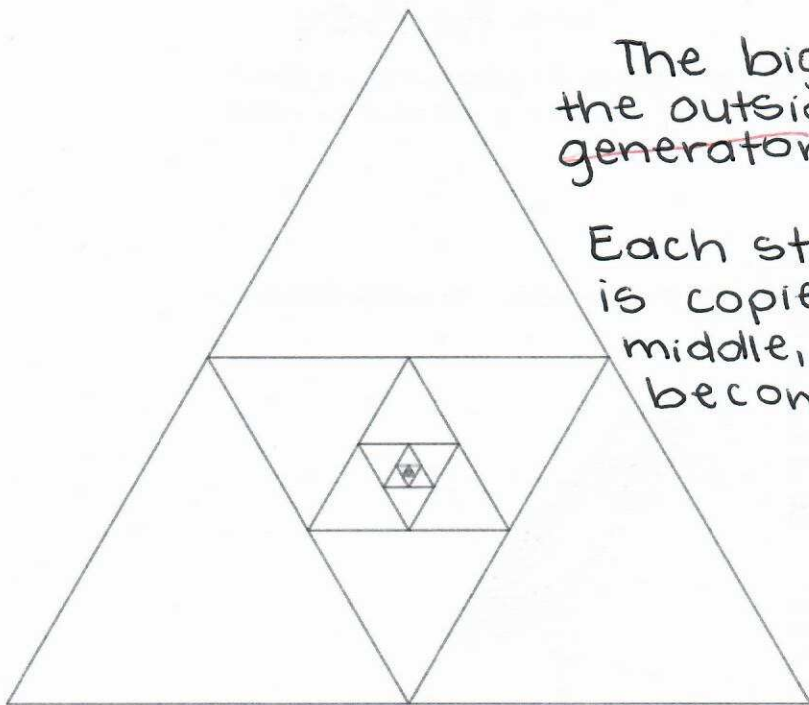
	# of Vertices	# of Edges	# of Faces	faces at each vertex	sides on each face
Tetrahedron	4	6	4	3	3
Cube	8	12	6	3	4
Octahedron	6	12	8	4	3
Dodecahedron	20	30	12	3	5
Icosahedron	12	30	20	5	3

Good work!

b. (8 pts) Which solids are duals, and how do we know from the table?

We can see from the table which solids are duals based off of their # of vertices and faces. These arrows point out which platonic solid matches with the other therefore showing its "twin!"

b. (5 pts) Start with an equilateral triangle. After many generations, the fractal looks like this:



The big triangle on the outside is the ~~an~~ generator. *initiator,*

Each step the triangle is copied, placed in the middle, rotates  $180^\circ$  and becomes smaller.



In the space above, describe the process by which this fractal is created: what is the generator; what does one do at each step?

**Problem 2: (25 pts) Platonic Solids**

a. (6 pts) Fill in the missing entries in this table:

	# of Vertices	# of Edges	# of Faces	faces at each vertex	sides on each face
Tetrahedron	4	6	4	3	3
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Dodecahedron	20	30	12	3	5
Icosahedron	12	30	20	5	3



b. (8 pts) Which solids are duals, and how do we know from the table?

tetrahedron is self dual ✓

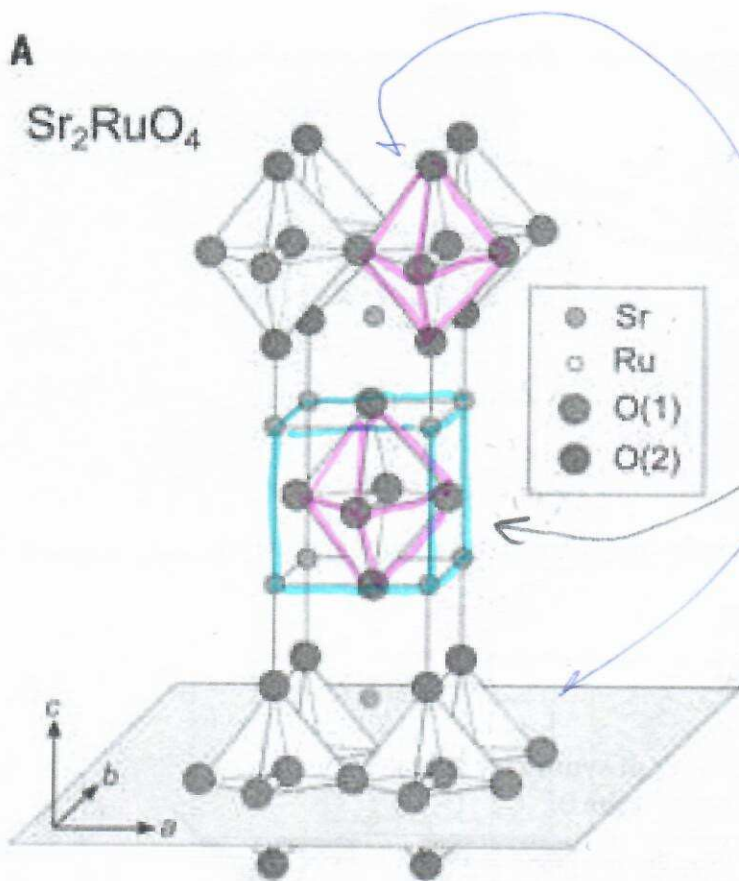
cube and octahedron are dual ✓

dodecahedron and icosahedron are dual ✓

I know because the number of faces at each vertex matches the number of sides on each face of the dual. ✓

c. (6 pts) Point out as many connections as you can to the Platonic solids in the molecule  $Sr_2RuO_4$

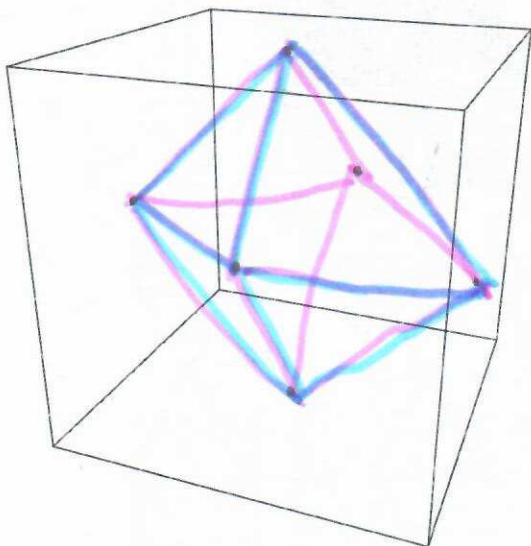
A



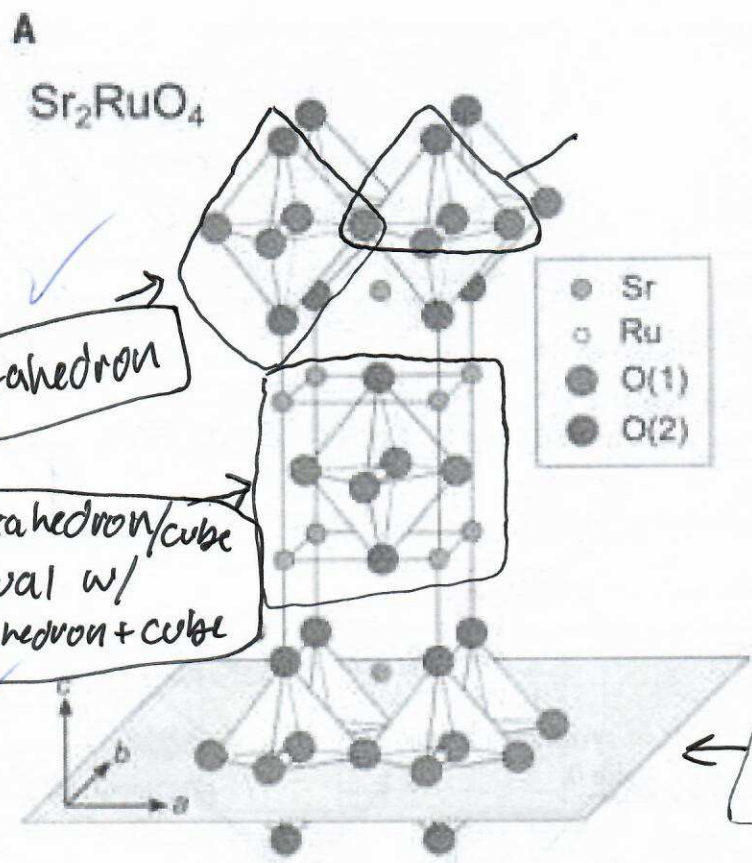
The molecule appears to be made up of cubes and octahedrons.

There is also an Octahedron inside a cube, demonstrating their duality

d. (5 pts) As accurately as possible, draw the dual Platonic solid into this cube (or any other cube of your choosing!):



c. (6 pts) Point out as many connections as you can to the Platonic solids in the molecule  $Sr_2RuO_4$

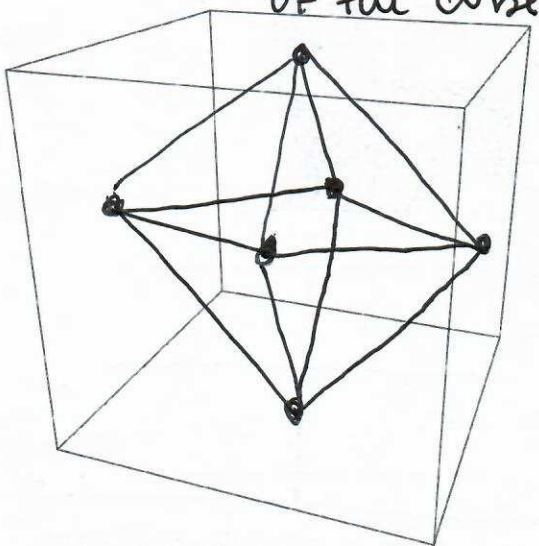


(although I cannot find an Icosahedron in this diagram it is used a lot in chemistry so I don't know if I am missing that here

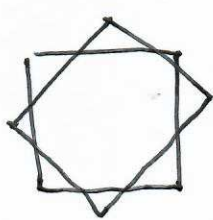
this is also a ~~flat~~ flat space 2D square

d. (5 pts) As accurately as possible, draw the dual Platonic solid into this cube (or any other cube of your choosing!):

each edge of the octahedron is in the face of the cube making it a dual + twin (also has one-to-one correspondence of each edge)



here's the duality aspect with a cube in the Octahedron just for fun ;)



Thanks!

**Problem 3: (25 pts) Symmetry**

a. (9 pts) Below each local logo, describe the symmetry you observe. For extra credit (up to 5 pts), correctly identify any knots or links within the logos.

*mobius band*



Rotational Symmetry 0:4



Reflection Symmetry  
Order 8 Rotational Symmetry



Rotational Symmetry 0:2

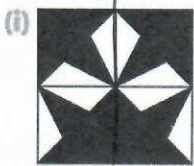
b. (16 pts) Carry out this exercise, but in T2(a) also include the order of rotational symmetry.

T2 Judith has lots of tiles, all like this one.

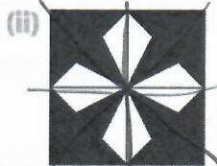


(a) Judith makes these patterns.

For each pattern, write down the number of lines of symmetry it has. If the pattern does not have reflection symmetry, write 0.



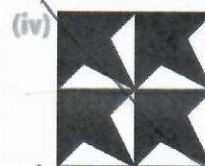
(i) 0:1 L:1



(ii) 0:4 L:4



(iii) 0:1 L:0

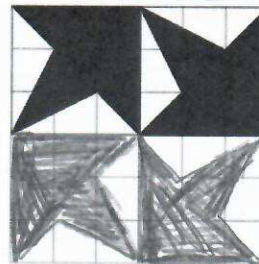


(iv) 0:1 L:1



(v) 0:4 L:2

(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.

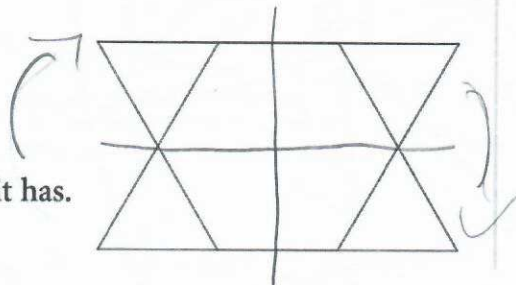


OCR

T3 This design is made from a regular hexagon and four equilateral triangles.

(a) What is its order of rotation symmetry?

(b) Write down the number of lines of symmetry it has.



(a) order of rotation = 2

(b) lines of symmetry = 2

Good work!



**Problem 3: (25 pts) Symmetry**

a. (9 pts) Below each local logo, describe the symmetry you observe. For extra credit (up to 5 pts), correctly identify any knots or links within the logos.



+3

Order 4 rotational

Solomon's knot link ✓



gravity  
IT RESOURCES ✓

Order 8

Rotational

Cinquantail Knot ✓



order 2 rotational ✓

Mobius

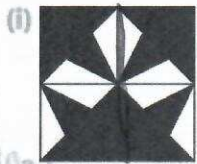
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T2 Judith has lots of tiles, all like this one.

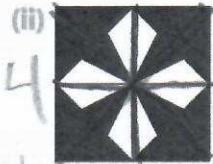


(a) Judith makes these patterns.

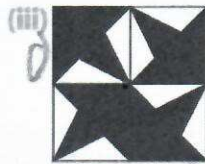
For each pattern, write down the number of lines of symmetry it has. If the pattern does not have reflection symmetry, write 0.



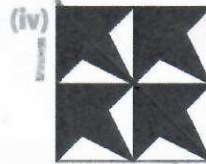
Order 1 Rotational



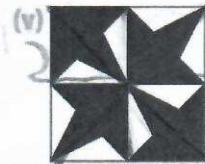
order 4 rotational



order 1 rotational

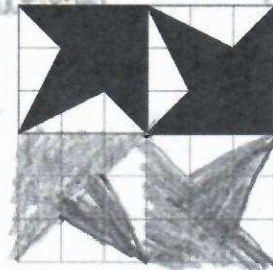


order 1



order 2

(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.

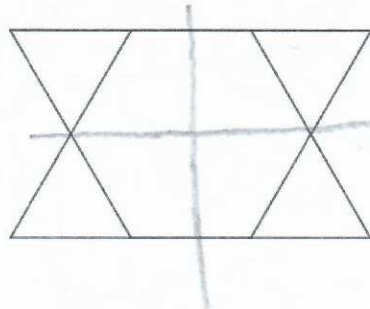


OCR

How would you tell?  
It's an unknot, at least.

T3 This design is made from a regular hexagon and four equilateral triangles.

- (a) What is its order of rotation symmetry?
- (b) Write down the number of lines of symmetry it has.



a: Order 2 rotational ✓

b: 2 lines of symmetry ✓

Problem 4: (20 pts) **Wallpaper** - Classify the following two wallpapers using Polyá's scheme:

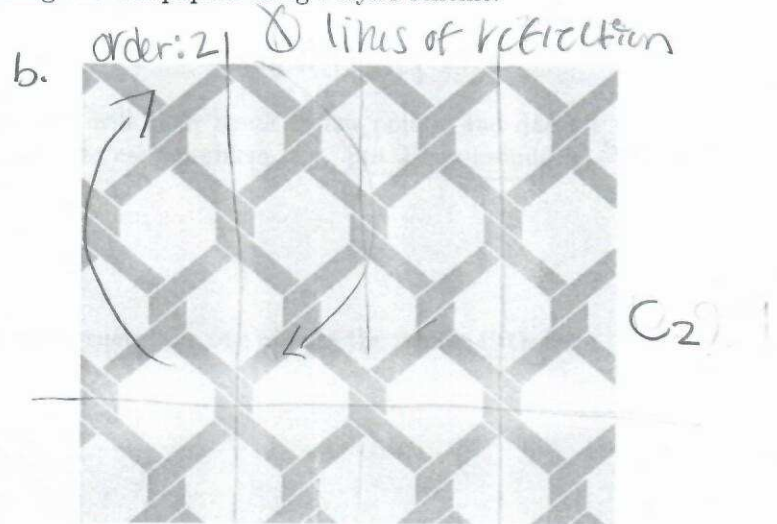


Figure 1: Propose legitimate candidates, discuss symmetries, etc. Then make your final choice.

a) is  $D_6$  because it has rotational symmetry order 6 and has reflection symmetry!

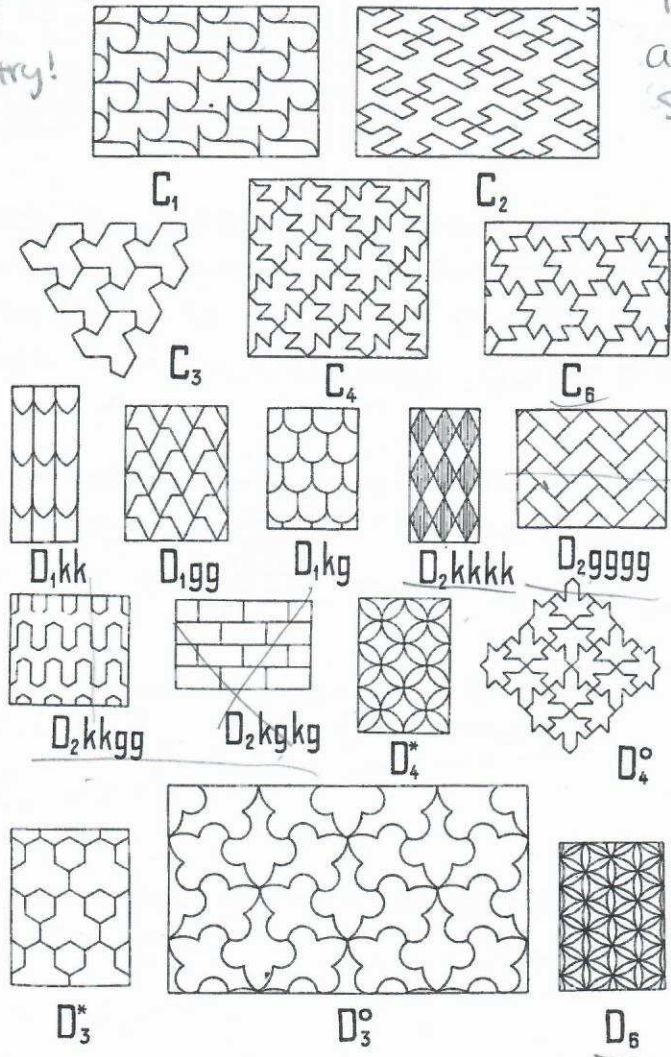
Über die Analogie der Kristallsymmetrie in der Ebene.

2S1

b) is  $C_2$ , because it has rotational symmetry order 2, and does not have reflection symmetry.

good reasoning

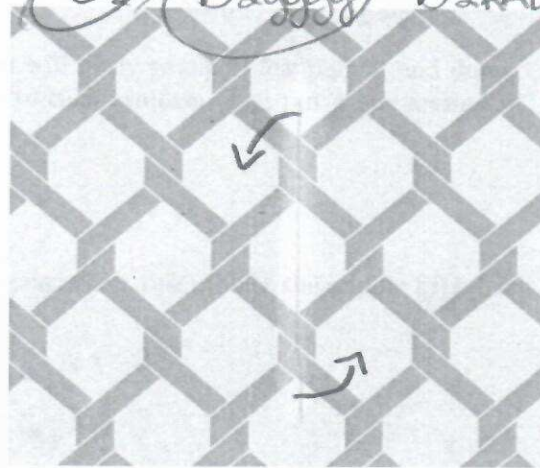
Good effort



Abi, you've got it!

Problem 4: (20 pts) Wallpaper – Classify the following two wallpapers using Pólya's scheme:

$C_3$   $C_4$   $D_6$

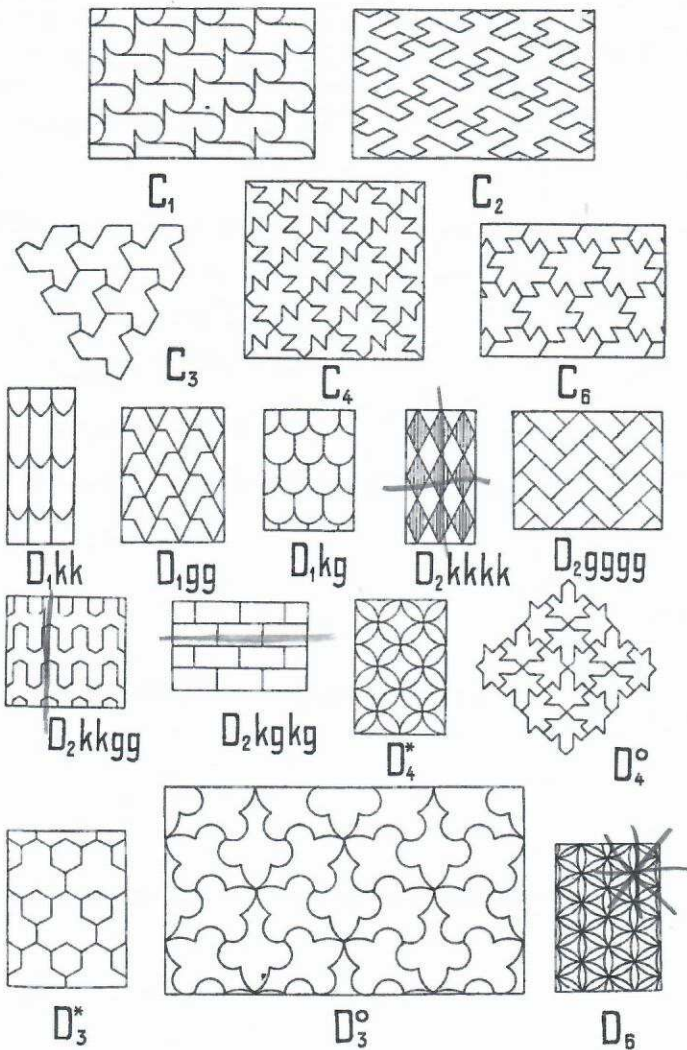


$D_2kkkk$   
 $D_2kgkg$

$C_2$   $D_2gggg$   $D_2kkgg$

Figure 1: Propose legitimate candidates, discuss symmetries, etc. Then make your final choice.

Ober die Analogie der Kristallsymmetrie in der Ebene. 281



brod is  
b ref, like  
 $D_6$

But the  
pattern is  
actually  
 $R_2!$

Either  $C_2$  or  
 $D_2gggg$ , likely  
 $C_2$  for number  
of sides in  
Hexagon shape.

mention no  
lines of  
symmetry –  
by which you've  
excluded  
 $D_2kkkk$ ,  $D_2kgkg$ ,  
+  $D_2kkkk$