MAT115 Quiz 12, Spring 2024

Name:

1. Give an example of the smallest infinity.

This will be a countable infinity - like the natural numbers, the evens, the Fibonacci #s, the primes, ...

2. (2 pts) There are 118 elements on the periodic table. We can think of the periodic table as a set containing 118 elements (which are actually elements!:). What's the size of its power set?

$$\text{Card}(\mathcal{P}(S)) = 2^{118}$$

Call the set S.

Card(S) = 118.

So

3. Describe the Hilbert Hotel.

The Hilbert Hotel has rooms numbered by the natural numbers: 1, 2, 3, ... So it's a countable infinity of rooms.

4. Which set is bigger – the natural numbers or the even natural numbers?

They're exactly the same size since they can be put in one-to-one correspondence:

$1 \leftrightarrow 2$
$2 \leftrightarrow 4$
$3 \leftrightarrow 6$
$\vdots \quad \vdots$

$n \leftrightarrow 2n$

5. Pascal's triangle floats on a sea of what?

A sea of zeros

0 0 1 0 0
0 0 1 1 0 0
0 1 2 1 0

6. (2 pts) Given the set A of artists: $\{Prince, Nirvana, The B52s\}$ (denote them $\{P, N, B\}$). Write down the power set of A, $P(A)$.

$P(A) =$

$\{\{\}, \{P\}, \{N\}, \{B\}, \{P,N\}, \{P,B\}, \{N,B\}, \{P,N,B\}\}$

$Card(P(A)) = 2^3 = 2^{Card(A)}$

7. (2 pts) Three infinite school buses show up to the Hilbert Hotel. Luckily it's empty: how do we slide them in?

In class, we sent each to powers of a prime:

Bus 1	Bus 2	Bus 3
$1 \leftrightarrow 2^1$	$1 \leftrightarrow 3^1$	$1 \leftrightarrow 5^1$
$2 \leftrightarrow 2^2$	$2 \leftrightarrow 3^2$	$2 \leftrightarrow 5^2$
$3 \leftrightarrow 2^3$	$3 \leftrightarrow 3^3$	$3 \leftrightarrow 5^3$
⋮	⋮	⋮

But there are other strategies!