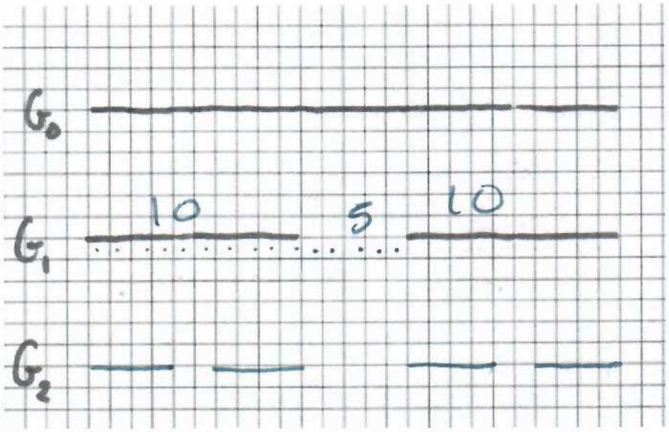
Directions: Show your work! Answers without justification will likely result in few points, and doesn't allow me to give partial credit. Indicate clearly your answer to each problem (e.g., put a box around it). Good luck! And remember to send me your knot photo....

Problem 1: (25 pts) **Fractals**

a. Consider this stick of (red) licorice of length 1: at each generation, one nibbles the middle **fifth** from each stick of licorice in the previous generation.



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2/5 1/5 2/5

i. (10 pts) Add the next stage of the fractal (generation 2, or G_2) to the figure above.

ii. (3 pts) How is the fractal's total length changing from step to step?

$G_0 - 1 - 4/5^0$
$G_1 - 4/5 = (4/5)^1$
$G_2 - (4/5)^2$

iii. (3 pts) How is the number of sticks increasing with each round?

$G_0 - 1 - 2^0$
$G_1 - 2 - 2^1$
$G_2 - 4 - 2^2$

In exponents of 2

iv. (2 pts) How many sticks will there be at generation G_6?

$2^6 = 64$

v. (2 pts) What will be the length of licorice at generation G_6?

$(4/5)^6$

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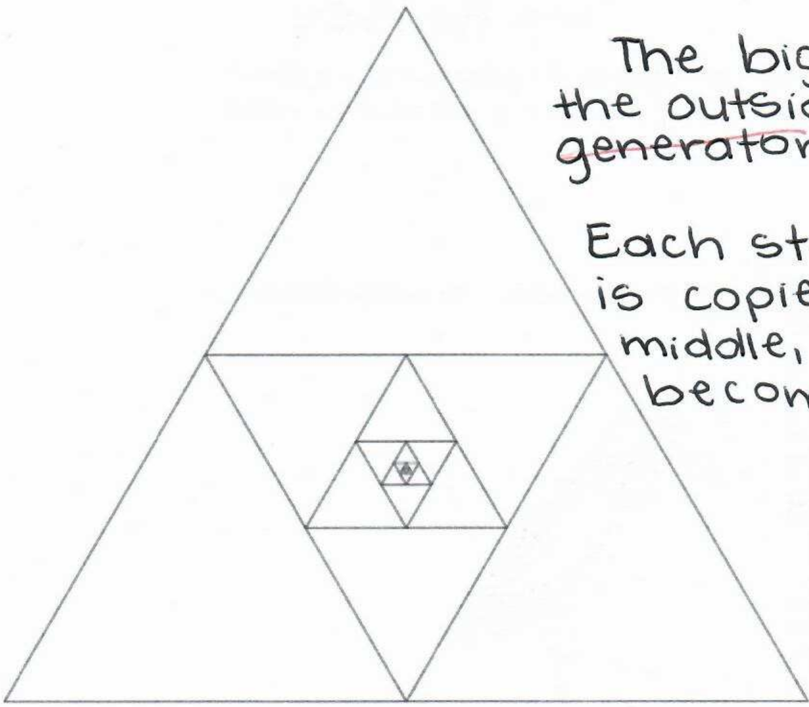
b. (5 pts) Start with an equilateral triangle. After many generations, the fractal looks like this:



The big triangle on the outside is the ~~generator~~. initiator,

Each step the triangle is copied, placed in the middle, rotates 180° and becomes smaller.

In the space above, describe the process by which this fractal is created: what is the generator; what does one do at each step?

Problem 2: (25 pts) **Platonic Solids**

a. (6 pts) Fill in the missing entries in this table:

	# of Vertices	# of Edges	# of Faces	faces at each vertex	sides on each face
Tetrahedron	4	6	4	3	3
Cube	8	12	6	3	4
Octahedron	6	12	8	4	3
Dodecahedron	20	30	12	3	5
Icosahedron	12	30	20	5	3

b. (8 pts) Which solids are duals, and how do we know from the table?

tetrahedron is self dual

cube and octahedron are dual

dodecahedron and icosahedron are dual

I know because the number of faces at each vertex matches the number of sides on each face of the dual.

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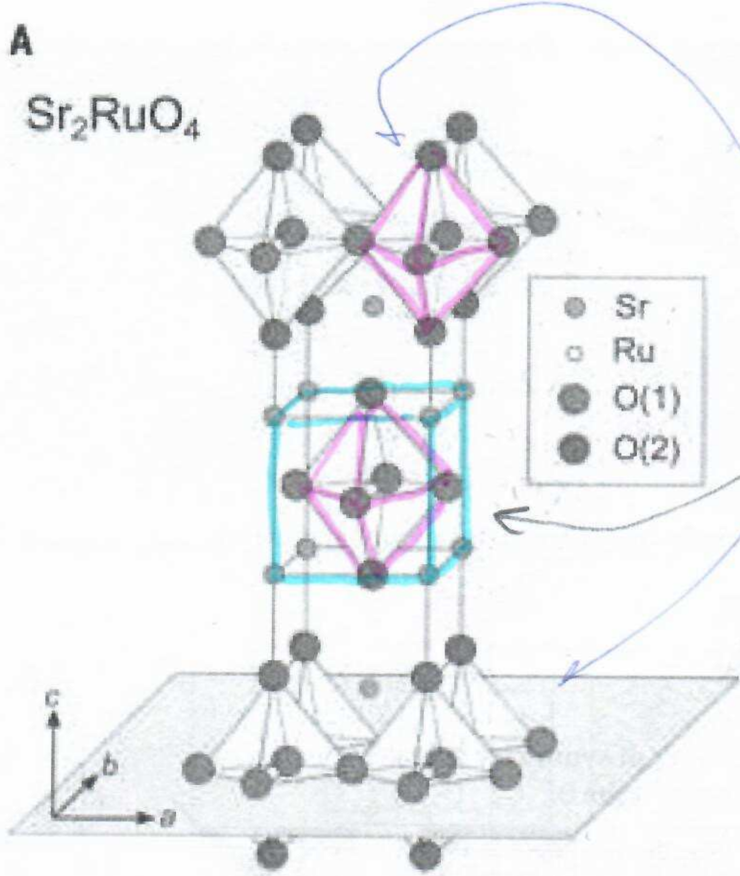
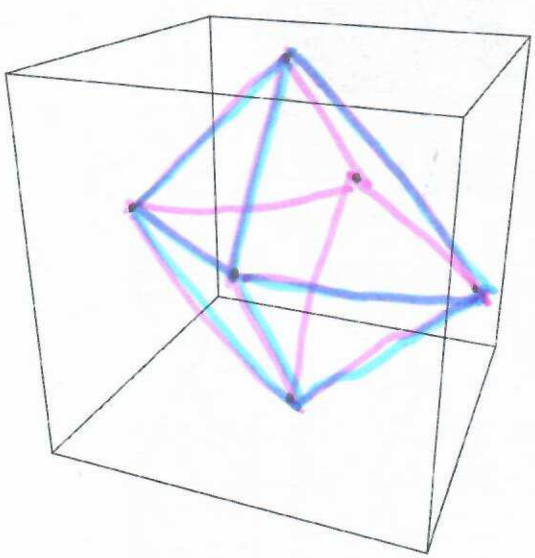
c. (6 pts) Point out as many connections as you can to the Platonic solids in the molecule Sr_2RuO_4



The molecule appears to be made up of cubes and octahedrons.

There is also an Octahedron inside a cube, demonstrating their duality

d. (5 pts) As accurately as possible, draw the dual Platonic solid into this cube (or any other cube of your choosing!):



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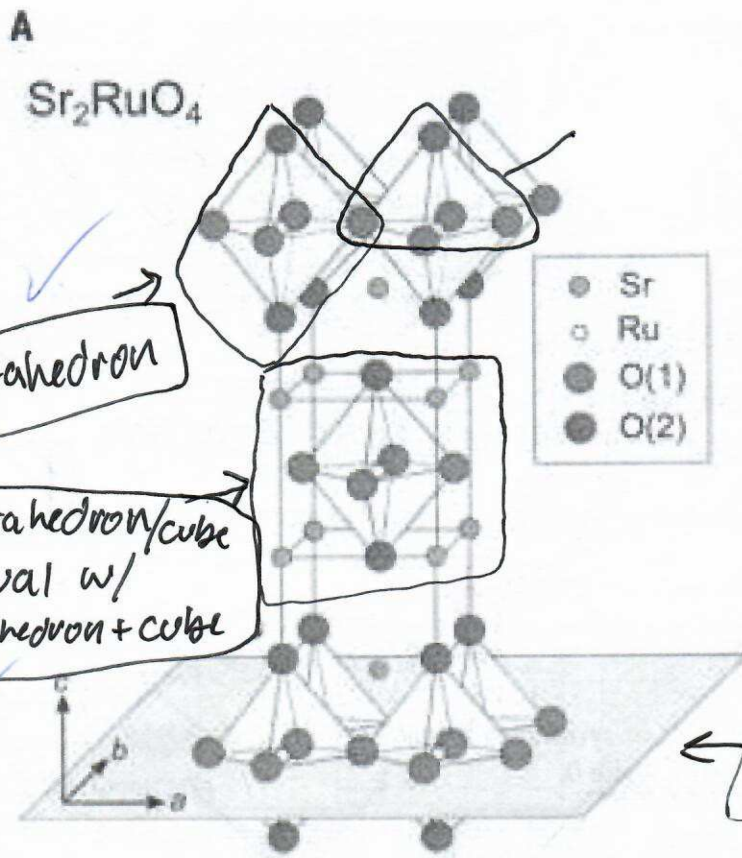


Octahedron

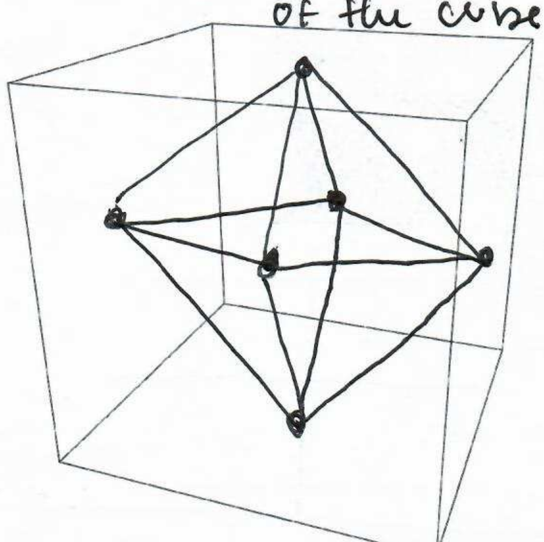
hexahedron/cube + dual w/ Octahedron + cube

although I cannot find an Icosahedron in this diagram it is used a lot in chemistry so I don't know if I am missing that here

this is also a ~~flat~~ flat space 2D square

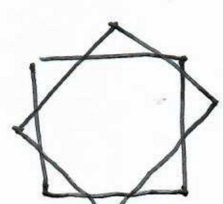
d. (5 pts) As accurately as possible, draw the dual Platonic solid into this cube (or any other cube of your choosing!):

each edge of the octrahedron is in the face of the cube making it a dual + twin (also has one-to-one correspondence of each edge)



heres the duality aspect with a cube in the Octahedron just for fun :)

Thanks!



Problem 3: (25 pts) **Symmetry**

a. (9 pts) Below each local logo, describe the symmetry you observe. For extra credit (up to 5 pts), correctly identify any knots or links within the logos.

{~~mobius band~~}

Rotational symmetry O:4

Reflection symmetry Order 8 Rotation symmetry

Rotational symmetry O:2

b. (16 pts) Carry out this exercise, but in T2(a) also **include the order of rotational symmetry**.

T2 Judith has lots of tiles, all like this one.

(a) Judith makes these patterns.

Rotation O= order

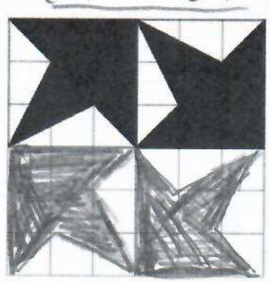
L = Lines of sym

For each pattern, write down the number of lines of symmetry it has. If the pattern does not have reflection symmetry, write 0.

(i) O:1 L:1 (ii) O:4 L:4 (iii) O:1 L:0 (iv) O:1 L:1 (v) O:4 L:2

Nice

(b) Copy and complete this tiling pattern so that it has rotation symmetry of order 4.

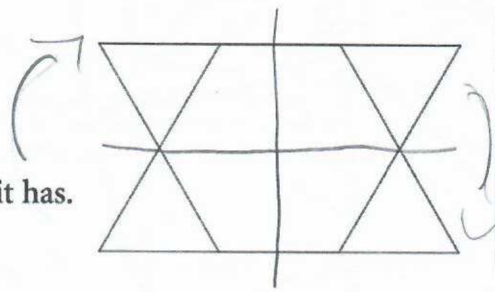


OCR

T3 This design is made from a regular hexagon and four equilateral triangles.

(a) What is its order of rotation symmetry?

(b) Write down the number of lines of symmetry it has.



(a) order of rotation = 2

(b) lines of symmetry = 2

Good work!

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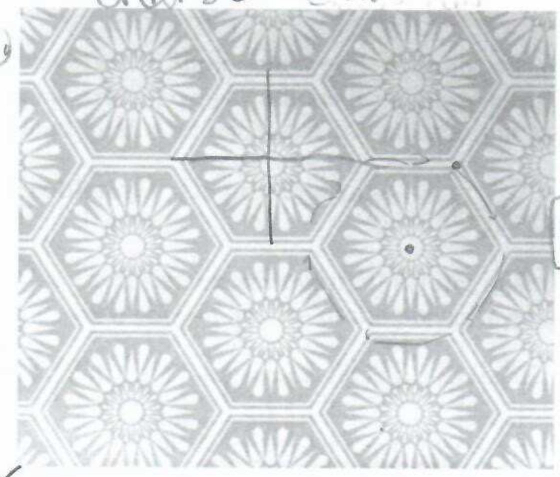
Problem 4: (20 pts) **Wallpaper** – Classify the following two wallpapers using Polya's scheme:

order: 6 has lines of sym def: * order: 2 0 lines of reflection

(a)



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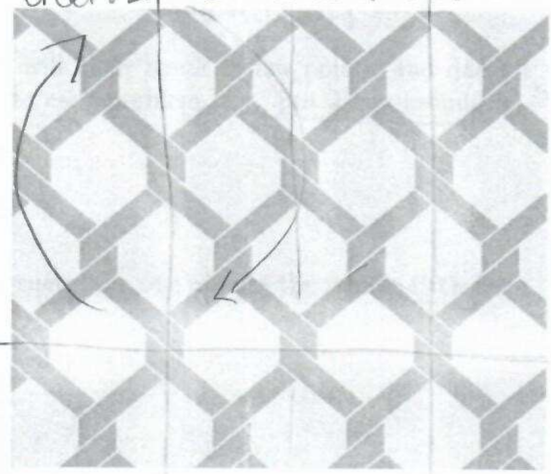
$\boxed{D_6}$

b.



C_2

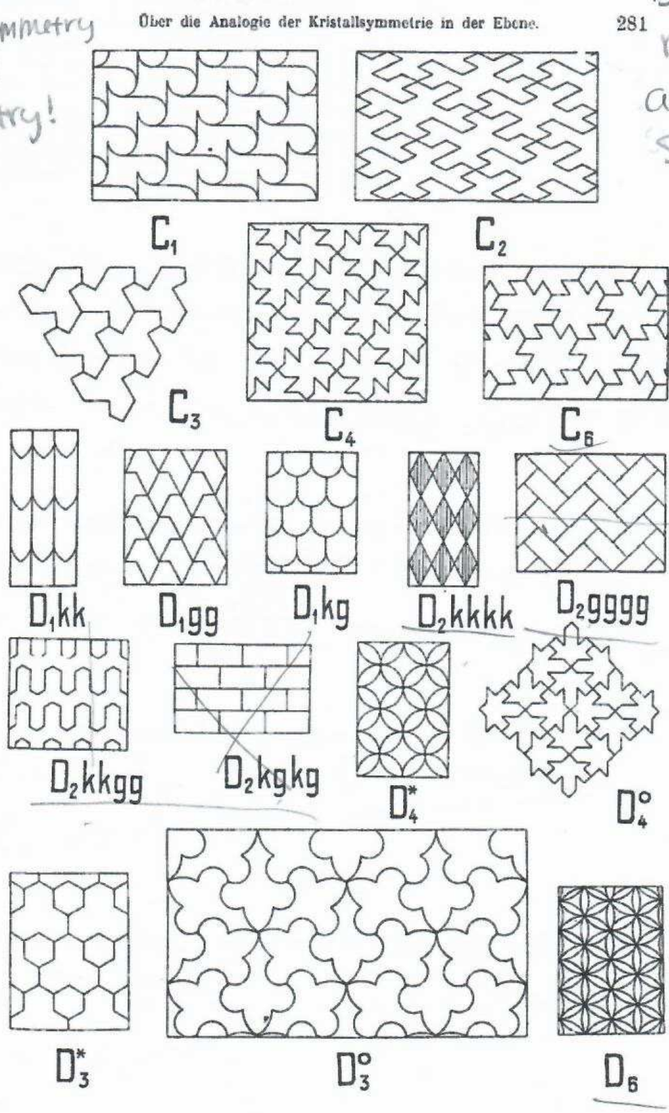
Figure 1: Propose legitimate candidates, discuss symmetries, etc. Then make your final choice.

a) is D_6 because it has rotational symmetry order 6 and has reflection symmetry!

good reasoning

b) is C_2, because it has rotational symmetry order 2, and does not have reflection symmetry

Good effort



Abi, you've got it! :)